

JTAR

EDITORS

GIL NAIZER

APRIL SANDERS

LAURA ISBELL

TAMI MORTON

SUSAN WILLIAMS



Journal of Teacher Action Research - Volume 5, Issue 3, 2019,
practicalteacherresearch.com, ISSN # 2332-2233

© JTAR. All Rights Reserved

JTAR

Journal of Teacher Action Research Volume 5, Issue 3, 2019

- | | |
|---|----|
| Maintaining Scientific Literacy in a Digital Age: The Transition of a High School Biology Class from Paper Textbooks to Digital Text Resources
Lisa Catalano Gizas | 4 |
| Examining the Effect of Language-Based Instructional Interventions in ELL and Non-ELL Language Production and Task-Oriented Behavior in Elementary Math, Science, and Social Studies Classrooms
Maggie Hoody
Joanna Yang Yowler
Michele Link-Valenstein
Anna Banti
Krystle Eilen
Andrea Saenz
Heidi Saari
Chris Pierret | 18 |
| Mathematics Stations in the Third Grade Classroom: Are They Worth It?
Rachel Perry | 38 |
| Surveying the Linguistic Needs of Team Members Traveling to Puerto Rico for a Short-Term Service Learning Trip
Megan A. DeVoss
Robert A. Griffin | 61 |
| Developing Critical Thinking, Justification, and Generalization Skills in Mathematics Through Socratic Questioning
Meighan Duffy
Manuela Heinz | 85 |
| Effective Mathematics Instruction for Native American Elementary Students: Strategies for Using Manipulatives
Traci Steigelmeier
Jarret D. Moore | 99 |



About the Journal

Founded in 2013, the Journal of Teacher Action Research (ISSN: 2332-2233) is a peer-reviewed online journal indexed with EBSCO that seeks practical research that can be implemented in Pre-Kindergarten through Post-Secondary classrooms. The primary function of this journal is to provide classroom teachers and researchers a means for sharing classroom practices.

The journal accepts articles for peer-review that describe classroom practice which positively impacts student learning. We define teacher action research as teachers (at all levels) studying their practice and/or their students' learning in a methodical way in order to inform classroom practice. Articles submitted to the journal should demonstrate an action research focus with intent to improve the author's practice.

Editorial Team

Co-Editors

Gilbert Naizer, Ph.D.
Texas A&M University-Commerce

April Sanders, Ph.D.
Spring Hill College

Associate Editors

Laura Isbell, Ph.D.
Texas A&M University-Commerce

Tami Morton, Ph.D.
Texas A&M University-Commerce

Susan Williams
Texas A&M University-Commerce

Production Editor and Webmaster

Chase Young, Ph.D.
Sam Houston State University

www.practicalteacherresearch.com

DEVELOPING CRITICAL THINKING, JUSTIFICATION, AND GENERALIZATION SKILLS IN MATHEMATICS THROUGH SOCRATIC QUESTIONING

Meighan Duffy

National University of Ireland Galway & Athlone Community College, Ireland

Manuela Heinz

National University of Ireland Galway, Ireland

Abstract This article reports on an action research study, which explored the impact of Socratic questioning on student learning in a second-level mathematics classroom in Ireland. While students engaged in a higher order mathematical task – the tower problem (Martino & Maher, 1999), the teacher used Socratic questioning techniques to challenge and support them to justify and generalize the problem as well as their thinking processes and solutions. The results of this study point to strong links between strategic Socratic questioning and students' involvement in critical thinking, justification, and generalization.

Keywords: teacher action research, Socratic questioning, mathematics education, justifications, generalizations, higher order thinking

Note: *This action research study was conducted by Meighan Duffy during the final year of her Bachelor in Mathematics and Education programme at the National University of Ireland Galway. Manuela Heinz acted as Meighan's research supervisor, supporting her throughout the development, implementation and writing of the research and this paper.*

Introduction

To question well is to teach well. In the skilful use of the question, more than anything else, lies the fine art of teaching (De Garmo, 1911)

This article will explore the area of questioning in the mathematics classroom. As a preservice mathematics teacher, I¹ have begun to analyze my questioning practices. From my observations of mathematics teachers during my initial teacher education, I have noticed a strong emphasis on repetition, convergent one-answer thinking and drill like procedures.

A variety of research studies indicate that mathematics teachers are not particularly adept at asking questions (Aizikovitsh-Udi, 2013). Watson and Young (1986) found that teachers ask as many as 50,000 questions a year while their students ask as few as 10 each (as cited by Vacc, 1993). As well as that, “about 60% of teachers’ questions require students to recall facts, about 20% require students to think, and the remaining 20% are procedural” (Gall, 1970, p. 713).

It is argued that the over emphasis on covering material as opposed to engaging students’ thinking is a result of teachers not fully appreciating the role of questioning in the development of subject knowledge. Many teachers assume that answers can be taught separately from questions (Elder & Paul, 1998). As a preservice teacher, I can relate to this misconception. On numerous occasions I have reflected on lessons and found my use of questioning to be very superficial and, at times, meaningless.

This action research study was inspired by my desire as an educator to help students to reach their fullest potential by making mathematics meaningful and relevant to their lives and interests. In order to fulfil my hopes and philosophy of education, I realize the importance of examining the types of questions I ask in the classroom and the educational objectives they can help my students and I to achieve. I believe that a greater understanding of questioning can allow me to encourage critical thinking amongst my students, thus making me a better educator.

In my research, I explored how I could use Socratic questions to enhance my students’ critical thinking, generalization and justification skills in the mathematics classroom.

Literature Review

Classroom questions have been classified in many different ways by various researchers. According to Gall (1970) there are at least 11 classifications of question types. However, mathematics classroom questions can be simplified to fall into one of two overarching categories: lower cognitive questions and higher cognitive questions. Lower cognitive or lower order questions are predominately used to determine students’ ability to recall information previously read or taught by a teacher with answers generally predetermined and fixed. These questions are sometimes referred to as convergent questions and correspond to the level of ‘knowledge’ outlined in Bloom’s taxonomy (Winnie, 1979). Higher cognitive or higher order questions, on the other hand, encourage students to think past the simple literal answering of questions, engaging them deeply with what is being asked to extend their understanding. The responses associated with these questions coincide with

¹ Throughout this article, the first person pronoun “I” refers to Meighan who, at the time, implemented this study as a preservice teacher during her last block of school placement.

‘application’, ‘analysis’, ‘synthesis’, and ‘evaluation’ levels in Bloom’s taxonomy (Winnie, 1979).

A model of questioning that is based on the use of higher cognitive questions is the ‘Socratic Model of Questioning’. According to Paul and Elder (2007, p. 2), “the key to distinguishing Socratic questioning from questioning per se is that Socratic questioning is *systematic*, *disciplined*, and *deep*, and usually focuses on foundational concepts, principles, theories, issues, or problems”. Essentially, Socratic questioning or Socratic dialogue is about probing thinking at a deeper level. Cox and Griffith (2007) also emphasised the importance of integrating Socratic questions and identified six categories:

1. Getting Students to clarify their thinking: ‘Could you expand on that?’, ‘Why do you say that?’
2. Challenging students about assumptions: ‘Does that always happen?’, ‘Why do you think that application applies here?’, ‘Is this always the case?’
3. Evidence as a basis for argument: ‘What are the reasons behind your answer?’ ‘Why do you say that?’
4. Alternative viewpoints and perspectives: ‘Did anyone answer this differently?’
5. Implications and consequences: ‘What can you conclude from this proof?’, ‘How does effect?’
6. Question the Question: ‘Do you think that was a relevant/important question?’, ‘Why do you think I asked that question?’, ‘Which of your questions turned out to be most useful?’

Using classroom questions to promote justifications and generalizations. Davis et al. (1992) found that when students are given a problem to work on independently they begin by building their own representation and solution, and when they have achieved this, they are usually interested in the ideas of, and in communicating ideas with, others. Once students believe their result is valid, they are ready to justify and generalize their solution. This is when teacher intervention is crucial (Martino & Maher, 1999). Martino & Maher (1999) found that, in general, students do not naturally seek to build a proof or justify their findings. Rather, students usually believe that finding a solution is enough.

A very important factor when learning mathematics, or any subject, is making connections with knowledge already acquired. Questions that invite students to make mathematical connections and generalizations such as “Have you ever worked on a question like this before?” deepen the understanding and appreciation for the problem at hand as well as the subject overall. This type of questioning allows the teacher to support students to link prior knowledge with new problems and, thus, be actively involved in the construction of their knowledge. This approach is in line with constructivist educational theories, advocating students’ discovery of their own mathematical understanding so as to engage them in active knowledge construction (Cobb, 1994).

Action research can be defined as teacher inquiry into classroom practice with a purpose of improving classroom practice and seeking improved understanding of educational situations that arise (Feldman & Minstrell, 2000). It can be used as a self-assessment tool that assists teachers in identifying the needs, assessing the development processes and evaluating the

results of the changes they design and implement (Johnson, 1993). My interest in the impact of questioning came about early on in my teaching practice. I recognized the need for a better use of questioning in the mathematics classroom from both observing colleagues and evaluating my own practice. The mathematics curriculum in Ireland has undergone many changes in the last number of years and I recognized questioning as an important tool in teaching the new Project Maths syllabus.

Project Maths has been introduced as a new Maths syllabus for second-level schools in Ireland. It aims to improve levels of engagement among students and achievement overall by placing more emphasis on conceptual understanding as well as practical and contextualized application, rather than the previous practice of rote learning (National Council of Curriculum and Assessment, 2012; O'Mahoney & Heinz, 2016).

After much reflection and evaluation, I decided that the focus of my questioning should be heavily linked with engaging students in critical thinking, specifically justifications and generalizations. Research in mathematics in the last decade has consistently called for the "need to promote student's learning that goes far beyond the acquisition of mathematical knowledge, but including also the development of mathematical capabilities such as problem solving, reasoning and communication" (Ponte, 2011 cited in Menezes et al., 2012, p.357). I recognized this need in the mathematics classroom and decided to act upon it by engaging in this study.

Methodology

This study was undertaken with a mixed ability transition year group (13-14 year olds) of 12 male students in a second-level single sex boys school in Ireland. At the time of undertaking this study, I completed my final school placement block as a student teacher in this school, and I taught this transition year² group twice a week.

During my classes in advance of this particular study, I gradually introduced the transition year group to questioning and discussion as a means to studying problems. It was something they were not accustomed to in the mathematics classroom previously. As part of each lesson, I encouraged students to talk about, discuss, and debate their solutions or thoughts about each task. I used the 'Socratic Model of Questioning' and, specifically, the questions outlined in the literature review (Cox & Griffith, 2007). I noticed that I used the 'why?' question most often.

² Transition Year is a one-year school programme that can be taken in the year after the Junior Certificate in Ireland. Students are approx. 15-16 years old. Depending on school population and funding it may not be available in some schools or compulsory in others. It is designed as a bridge between junior and senior cycle programmes and schools devise their own programmes.

During my ninth week teaching the transition year class, I initiated my specific research task – The Tower Problem (Martino & Maher 1999), which asked students to:

1. Build as many towers 4 cubes tall as possible with cubes of two colors
2. Figure out how to convince others that they had built all possible towers combining the cubes of two colours (that there were no duplicates and that they had not omitted any options).

Each student was provided with the problem sheet – explaining the task as well as a bag of cubes to allow students to build their towers (a sufficient amount for the 16 different towers that could be built as well as many extra cubes were provided to allow students to build duplicates). Extra paper for note taking was also provided.

While students worked on this task, the following data were collected:

- a voice recording of the full class
- students' written work
- researcher's observations and reflection notes.

All voice recordings were transcribed verbatim. Data analysis focused systematically on the relationship between the use of teacher questioning and the resulting student justifications and generalizations. Socratic questions used by the teacher as well as student generalizations and justifications were noted and categorized (see tables 1 and 2).

The limitations of this study are evident in the small number of students that took part, the fact that the school setting is a single-sex male school, and the main criteria for answering the research question relies heavily on one specific task.

Results

Questions that stimulated student justification and generalization. It is clear from the voice recorded data that there was a strong relationship between the questions students were asked and their progression with building a solution and working towards a justification. The questions student 1 was asked allowed him to take ownership of his solution. He was then, after several further questions, able to show how he built the towers and to use that as a justification for his solution. His explanation needed work and he was aware of that by the end of his interaction with the teacher, and he was then left to concentrate on developing his explanation. When questioning student 2 it was evident that deeper thought was needed around the construction of his towers in order to solve the problem. Strategic questioning allowed him to reflect on his methods and focus on those to build the remainder of his solution. The conversation with student 3 shows again the importance of questioning. This student built 15 towers and believed he had a solution but when questioned on how he knew he had all the possible outcomes he re-considered and realized further work was needed.

The voice recorded data further shows a direct relationship between the questions students were asked and their extended efforts towards generalizing their justifications. When questioning student 4, it was clear that he was convinced that he had fully justified his solution. Further questioning engaged him in more critical thinking and motivated him to prove his solution for 3 towers. The transcript conversation with student 5 is particularly interesting. His solution of 2^4 was correct, but it was evident that, when questioned, his knowledge of this fundamental principle of counting formula was limited. Although 2^4 and 4^2 worked out the same for the number of combinations in this particular problem they would not for towers of a different height. Instead of correcting the student, strategic questioning put him on the path of discovering that for himself.

The extracts provided below serve to provide an authentic flavor of the student-teacher interactions and the use of different types of questions. Socratic questions formulated by the teacher to motivate students to keep trying, to generalize and to justify are highlighted in bold print.

Justification

Student 1: Have 17 but don't think I'm right, think I've an extra one.

Teacher: Do you? Can you find it?

Student 1: Yep. There.

Teacher: Where's that one?

Student 1: Oh it's there, no it's not, [pause] there it is.

Teacher: Do you think you have them all now?

Student 1: Yea [pause] think so.

Teacher: You've no more extras?

Student 1: No. Don't think so.

Teacher: **Ok, c'mon you have to be sure.**

Student: Yea I am sure, cos I have all the possible outcomes. I have one on top, one in the middle and then all the different outcomes.

Teacher: Right?

Student: Em.. [student pauses and studies his built towers]

Teacher: **What way could you explain it to someone to prove you definitely have all the outcomes?**

Student 1: Em.. [long pause] I started with them all green and then I put one in place for each of the greens, then I did the same with the blue [pause] and eh, I got all the ways of one of an odd colour in the four of them, you work out [pause] change up the different colours as many times as you can

Teacher: **Okay, so think about a way you can write that down. I think you're on to something there.**

Teacher: **How would you describe your pattern?**

Student 2: What pattern?

Teacher: Your system of doing these.

Student 2: Em..I duno [student pauses and studies his built towers]

Teacher: **Would you describe it any way at all? Is it just a bit random?**

Student 2: No, like you always continue down from the simplest one here..[student demonstrates with his built towers] you get 8 here...colours are always touching.

Teacher: Okay, and how many do you have?

Student: 8..[counting] no, 13.

Teacher: **Do you think you have them all now?**

Student 2: No.

Teacher: How many do you think there are?

Student 2: I'd say there are 22 or 24 all together.

Teacher: You think?

Student 2: Yea about that.

Teacher: Okay, keep going.

Teacher: How many do you have?

Student 3: I have 15.

Teacher: And how many do you think there are in total?

Student 3: 15.

Teacher: Do you?

Student 3: Well I can do them like and see.

Teacher: **How do you know you haven't missed one or made one twice?**

Student 3: I don't.

Teacher: No?

Student 3: Well I'm pretty sure I haven't like.

Teacher: **Okay, well how about you take another look and I'll come back to you when you are sure.**

Generalization

Teacher: What was your system?

Student 4: Well start off 4 colours, 3 colours, 2 colours in each and then one. And that was my way.

Teacher: **What about 3 cubes tall?**

Student 4: [pause] Emm..Wouldn't be as much outcomes.

Teacher: **Why do you think that?**

Student 4: Cos there is less blocks and that would take out some.

Teacher: So how many outcomes would you reckon?

Student 4: It would be 9.

Teacher: You think?

Student 4: Yea, going by the same way.

Teacher: **Ok try it.**

Teacher: Why is it 2^4 ?

Student 5: Because 2 different colours and 4.

Teacher: **Where did your formula come from?**

Student 5: Just the numbers 2 and 4.

Teacher: How did you know to do that?

Student 5: Well you can put 2^4 or 4^2 , same **answer 16**.

Teacher: **Ok** well would it work for 3 towers?

Student 5: Emm..I duno you'd probably have to change it.

Teacher: How many outcomes do you think you'd get using that formula?

Student 5: 9.

Teacher: Think so?

Student 5: Yea probably.

Teacher: **Okay, try it**

Extent of use of Socratic Questioning. As part of the analysis of transcripts, all Socratic questions asked by the teacher were counted and categorized. Table 1 provides an overview of the number of Socratic questions used by the teacher by question category. It shows that the majority of teacher questions (86) served to encourage students to clarify their thinking. The voice recording also provided evidence that students were frequently challenged to test their assumptions (38 questions) and/or to provide evidence for their argument (32 questions). Questions encouraging students to consider alternative viewpoints or implications, and questions exploring questions were also used but to a lesser extent (between 9 and 20).

Table 1: The Type and Corresponding Amounts of Socratic Questions Used

Socratic Questioning Type	Examples of Most Common Questions Used
Getting students to clarify their thinking (approximately 86 questions of this type)	Why do you think that? Do you think you have them all? What was your system?
Challenging students about assumptions (approximately 38 questions of this type)	Are you sure you have them all? Do you think that's true for all towers? What about 5 towers tall / 3 towers tall?
Evidence as a basis for argument (approximately 32 questions of this type)	How could you convince someone you definitely have them all? Have you thought of a way you'd prove it?
Alternative viewpoints and perspectives (approximately 11 questions of this type)	What do you think about this [student name]? How about if you compare these two?
Implications and consequences (approximately 17 questions of this type)	Do you think they'd be convinced with your proof? Do you need to rethink that a little then?
Question the question (approximately 9 questions of this type)	Well, what do you think? Do you think they are the same?

Extent of student justifications and generalizations. The transcript and students' written work were analyzed to establish the number of students providing justifications and/or generalizations (see Table 2).

Table 2: Justifications and Generalizations (n=12)

Number of students providing justifications (Proof by cases, Staircase Proof, Proof by opposites)	Justifying by generalizing - using previous knowledge to prove this problem (2^n & Tree Diagrams)	Generalizing the justification (Applying justification to towers of different heights – 3 tall, 5 tall)
12	13	8

Justifications: In the class of 12, all students had, by the end of the class, justified their 16 towers, 4 tall with two colors. The most commonly used explanation of their solution was case by case – all towers with just one of the colors, all of the towers with just two of one of the colors, etc. Some students proved the problem by explaining their visualization of the towers as a stair pattern and others found a tower and found its opposite until they could find no more.

Insights from teacher reflections. The analysis of teacher reflections resulted in three core insights:

1. Student's urge to find a solution quickly

2. Difficulties with recognizing general applicability of formula
3. Socratic questioning modeled by the teacher encouraged peer assessment

Students' urge to find a solution quickly. Something that I had not anticipated was students' urge to find a solution to the problem before even arranging their combinations (building the towers). Students were trying to apply their somewhat limited knowledge of the fundamental principal of counting to the task by using the number of cubes tall (4) and the number of colors (2) – "Is it $4 \times 4 \times 4 \times 4 \times 2$? $m \times n$ or something." I dealt with this by continuously telling students that the answer was up to them to decide and encouraged them to build the towers before making assumptions.

Difficulties with recognizing general applicability of formula. When students began to get the correct answer of 16 and I questioned them on how they knew they had them all and what was their system of finding all the combinations, all students were able to explain their methods – some explained proof by cases (towers of all one color, towers with 3 of one color, 2 of each color etc), proof by opposites (finding a combination and then finding its opposite until all combinations are exhausted) and others explained a staircase proof (where they arranged the towers side by side to form a diagonal for the different cases).

When I further questioned the students on the way in which they would choose to prove it, very few recognized their explanation as a type of proof. One student who had a solution as well as two justifications asked "Miss, are you going to give us the answer at the end or what's the story?" Some decided a tree diagram would be best, most tried to apply the fundamental principal of counting (2^n) to the answer they got, but it was clear that none of them could clearly define or apply the fundamental principle of counting to begin with. When questioned on where the formula came from one student said "Text and Tests - the blue one." It was only when I introduced the generalisation questions such as: "Would it work for any towers? What about 3 tall? How many towers do you think you would get if I said to build them 5 tall?" that students recognized the general applicability of the formula.

Socratic questioning modelled by the teacher encouraged peer assessment. After all students had had sufficient time to articulate an answer and begin the process of justification the noise levels in the room began to rise. I found that after I had circulated around the room and asked the majority of students questions that caused them to reflect and reorganize their solutions, the students themselves began to critically assess each others' work. They all seemed to stick with their own original methods but became very interested in the ideas of their peers.

Justifying by generalizing and generalizations. All of the students made further attempts to prove their solution by applying their previously acquired knowledge of probability. The majority recognized the fundamental principle of counting as it applied to this problem but had to then generalize that further to ensure it would work for towers of any height. Many students also used their squares or drew out a tree diagram to solidify their solutions. The figures show that there were 21 types of generalizations altogether, which demonstrates just how many different angles many students took to prove their problems.

Discussion

The purpose of this research study was to determine how I can use Socratic questions to enhance students' critical thinking, generalization and justification skills in my mathematics classroom.

Maher and Martino's (1999) observation that students do not naturally seek to build a proof or justify their findings was clear to see from the observation notes as well as the transcripts. Socratic questioning proved an important driving force in motivating students to continue to search for and critically assess their solutions. Students were not accustomed to this type of continued follow-up questioning in their typical mathematics classes and neither was I, their teacher. The data clearly demonstrates that students' learning has been significantly deepened through the use of Socratic questioning which challenged them to think critically, experiment, justify and generalize. This active engagement opened up many opportunities for constructivism in mathematics – students' discovering their own mathematics (Cobb, 1994).

The findings from the state exams over the last number of years with regards to higher order skills (Jeffes et al., 2012) have evidenced students' urge to apply formulas without critical thought. Students in this study showed inexperience with the communication of mathematics and a lack of confidence in their solutions. The analysis has shown that students were generally used to, and expecting, one answer only problems; despite the various valid justifications and generalizations worked out by themselves, they still assumed that there existed one 'best solution' to the problem.

Overall the findings show just how central classroom questioning is in encouraging and supporting students to justify and generalize in mathematics. In a mixed ability class of twelve students, every student arrived at, and justified, the correct solution, thirteen further justifications through generalizations were made as well as eight solid justifications for towers of all heights (see Table 2).

I began this action research with concerns about my use of questioning. I not only studied the different types of questions but also the outcomes I wanted to achieve as a result of them. I found that students lacked practice in justifying and generalizing their solutions in mathematics. The reliance on the textbook and convergent one answer thinking was evident, and from the literature, I was aware that an appropriate way of enhancing students' skills in these areas was through questioning. Extensive reflection and evaluation made me realise the importance of listening to students and of clarifying their thinking before constructing questions. The 'Socratic Model of Questioning' proved an important tool in self assessing and guiding my use of questioning throughout my teaching practice.

Despite the limitations of the study, I believe the findings demonstrate a strong relationship between the use of Socratic questioning and students' effort and ability to engage in justifications and generalizations of solutions. The importance of careful monitoring of

students' progress, knowing when to probe and when to step away is evident from the qualitative data provided in the excerpts.

The central focus of action research is the cycle of self-evaluation and learning. Before the implementation, I was nervous and unsure about how I was going to handle the mixed ability in this context, how students would react to the problem, and if I would be able to remember all the questions I wanted to ask. I was, however, surprised very early on about how closely the progression of the class matched the literature that I had reviewed beforehand. Although I had a list of prepared questions with me, I did not need to look at them during the class. My research and preparation gave me the confidence to listen to my students, assess their progress and question accordingly.

Conclusion

This research project has given me great hope for my career in teaching mathematics. If students can achieve this level of critical thinking and create that many justifications and generalizations as a result of the use of Socratic Questioning on one task, then what could they achieve over a year? The findings of this study have encouraged me even more to focus on and practice Socratic Questioning to enhance my students' critical thinking, generalization and justification skills in the mathematics classroom. Now, more than ever, with the introduction of the new Project Mathematics syllabus, it is of paramount importance that, as a teacher, I enhance students' critical thinking, justification and generalization skills. It is clear that the questioning strategies used in this study have the power to do just that.

About the Authors

Meighan Duffy is a former student on the Bachelor in Mathematics and Education programme (for second-level teaching) in NUI Galway and is now in her fifth year teaching Mathematics and Applied Mathematics in Athlone Community College. Email: meighanduffy@yahoo.com

Manuela Heinz, Ph.D. is a Lecturer and the Director of the Professional Master of Education at the National University of Ireland Galway. She has a strong interest in collaborative educational action research and supported with the development and implementation of this study as a research supervisor. Email: manuela.heinz@nuigalway.ie

References

- Aizikovitch-Udi, A., Clarke, D., & Star, J. (2013). Good questions or good questioning: An essential issue for effective teaching. In *CERME8: 8th Congress of the European Society for Research in Mathematics Education*. Antalya, Turkey.
- Cobb, P. (1994). Constructivism in mathematics and science education. *Educational Researcher*, 23, 4-4.
- Cox, S. & Griffith, A. (2007). *'Outstanding teaching'*. Osiris Educational.
- Davis, R. B., Maher, C. A., & Martino, A. M. (1992). Using videotapes to study the construction of mathematical knowledge by individual children working in groups. *Journal of Science Education and Technology*, 1(3), 177-189.
- DeGarmo (1911) *Interest and attention*. New York: Macmillan.
- Interest and Education. The Doctrine of Interest and its Concrete Application.
By Prof. C. DeGarmo. Pp. xiii + 226. (New York: The Macmillan Company, 1902.) Price 4s. 6d. net.
- Elder, L., & Paul, R. (1998). The role of Socratic questioning in thinking, teaching, and learning. *The Clearing House*, 71(5), 297-301.
- Etemadzadeh, A., Seifi, S., & Far, H. R. (2013). The role of questioning technique in developing thinking skills: The ongoing effect on writing skill. *Procedia-Social and Behavioral Sciences*, 70, 1024-1031.
- Gall, M. D. (1970). The use of questions in teaching. *Review of educational research*, 40(5), 707-721.
- Feldman, A., & Minstrell, J. (2000). Action research as a research methodology for the study of the teaching and learning of science. *Handbook of research design in mathematics and science education*, 429-455.
- Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheeler, R., & Dawson, A. (2013). Research into the impact of Project Maths on student achievement, learning and motivation. *Slough: National Foundation for Educational Research*, 1-72.
- Johnson, B. (1993). Teacher-As-Researcher. ERIC Digest. Washington, DC: ERIC clearinghouse on languages and linguistics.(ERIC Document Reproduction Services No. ED355205). Retrieved November, 1, 2004.
- Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior*, 18(1), 53-78.
- Menezes, L., Oliveira, H., & Canavarro, A. P. (2012). Teacher practice in an inquiry-based mathematics classroom. *International Journal for Mathematics in Education*, 4, 357-362.

National Council for Curriculum and Assessment (2012) Junior Certificate Mathematics Syllabus. Dublin: Stationery Office.

O'Mahoney, K & Heinz, M. (2016) 'Using Comment Only Marking in a Mathematics Classroom in the Republic of Ireland: Experience and Learning of a Student Teacher'. *Journal of Teacher Action Research*, 2(2):58-69.

Paul, R. & Elder, L. (2007). *The Thinker's Guide to the Art of Socratic Questioning*. The Foundation of Critical Thinking. Available at: https://www.criticalthinking.org/store/get_file.php?inventories_id=231&inventories_files_id=374. Accessed on 17/05/2019.

Newton, L. D. (2013). Teachers' Questions: Can they support understanding and higher-level thinking?. *Research journal. Journal de recherches.*, 1, 6-17.

Winne, P. H. (1979). Experiments relating teachers' use of higher cognitive questions to student achievement. *Review of Educational Research*, 49(1), 13-49.

Vacc, N. N. (1993). Implementing the 'professional standards for teaching mathematics': questioning in the mathematics classroom. *Arithmetic Teacher*, 41(2), 88-92.