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DIFFERENTIATING INSTRUCTION TO INCREASE CONCEPTUAL UNDERSTANDING AND ENGAGEMENT IN MATHEMATICS

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Abstract This action-research project aimed to explore several strategies that teachers can use to develop conceptual mathematical understanding and increase behavioral engagement for students with differing instructional needs. Specifically, I investigated strategies for differentiating instruction, including individualized instruction in flexible groups and the use of problems with multiple entry points over a five-month period in a 6th grade classroom. Analyses focused on six focal students, all of whom were English learners or former English learners. Overall, findings suggest that the use of flexible instructional groups and math problems with multiple entry points can help teachers meet the varied needs of students, thus allowing all students to be more engaged and successful in the classroom.

Keywords: differentiated instruction, English learners, flexible small groups, mathematics, conceptual understanding, student engagement

Introduction

I cringe as I remember my very first day as a classroom teacher. It was a refreshing August morning as I eagerly delivered my 6th grade math lesson. I had meticulously created a PowerPoint presentation on place value, complete with colorful fonts and engaging animations. I delivered my carefully crafted lesson with stereotypical first-year-teacher enthusiasm and handed out a worksheet for independent practice. My students, with their own first-day-of-school motivation, diligently began completing the problems. Gradually, students began to raise their hands with questions. I raced through the rows of students, literally jogging from desk to desk trying to lend support and answer questions. Some students looked at me with embarrassment, not having been able to start the first problem on their own. Others reached a moment of panic as soon as they saw a word problem. While I struggled to offer enough support to the students who needed it, several other students raced through the problems and looked at me with eager eyes, asking, "What do we do next?!" I realized I didn't have an answer for them. For some students, the problems were not challenging enough to extend their thinking, yet others had barely started. I wondered, how could I be an effective teacher to both of these sets of learners? This is the question I endeavored to answer for myself, and for other teachers who face similar challenges.

Teachers often struggle with how to differentiate instruction in order to simultaneously meet the needs of all students in their class. However, whether or not to group students according to their current mathematics achievement has been a contentious issue in education since the 1980s (Boaler, 2013). In California, the average size of a public upper elementary school classroom is over 25 students, and about one third of these are still in the process of learning the language of instruction (CalEdFacts, 2014). It is inevitable that a single classroom will represent a wide variety of student ability levels, learning styles, strengths, and needs. Teachers, therefore, must be well equipped with strategies that allow them to maximize the academic and intellectual growth of all types of learners, including meeting the needs of English learners (ELs) and former ELs. We use the term English learners (Els) here because we believe this term is familiar to our readers. However, our beliefs are more in line with term *emerging bilingual* instead of English learner as a way of emphasizing the value of bilingualism (Garcia, 2009). In our own classrooms, my co-author and I have noticed students often show signs of disengagement when instruction does not match their current level of understanding, be it too easy or too difficult. When it comes to mathematics instruction, one size does not fit all. Receiving individualized support and guidance catered to their specific needs and strengths can allow all students to thrive academically in the classroom.

This action-research study aimed to uncover how two different strategies for differentiation can be used in a 6th grade classroom with the goal of increasing mathematical understanding and behavioral engagement. Although orchestrating several student groups at the same time can be challenging, we found it to be an effective way for one new teacher to meet the disparate needs of students, allowing us to use strategies suggested by past research to support ELs in particular. The following overarching questions guided our inquiry:

1. How can flexible, small group instruction impact the conceptual understanding and behavioral engagement of students with differing needs?
2. How can problems with multiple entry points (Low-Floor-High-Ceiling problems) serve as a way to differentiate instruction for students with differing needs?

Teacher-Researcher Positionality. The first author was the classroom teacher. At the time of data collection, I was in my first year of teaching. This project was conducted as part of my Masters of Education degree at a large, research-oriented university. I later extended the analyses of my project for publication. Unless otherwise noted, use of the pronoun "I" throughout the text refers to me, and "we" refers to both authors.

The second author is a former bilingual teacher and was the instructor of the two-quarter research methods course I attended as part of my Master's program. As course instructor, she guided the design of this inquiry project, encouraged me to share what I learned with other educators, and collaborated with me in writing this article.

Flexible Ability Grouping. Ability level grouping remains a controversial topic in education, and research on its impact remains mixed. While some research has demonstrated positive effects, other research suggests ability grouping can have a negative social impact on students (Hallam, Ireson, Mortimore, & Davies, 2000). In contrast, seminal work by Slavin (1987) demonstrates that ability level grouping can be an effective instructional strategy, as long as the ability groups are confined to a specific subject (e.g. math or reading). Slavin further suggests that ability grouping allows higher achieving students to be exposed to an appropriately accelerated pace of instruction, while allowing lower achieving students to receive more attention and practice.

Similarly, Gibbons (1991) discusses the notion of skills grouping: the arrangement of students into groups based on their ability levels and needs. Skills grouping, when done fluidly and for a short period of time, allows students to be exposed to instruction and content that match their current needs and level of understanding. Small-group instruction is defined as situations in which three or more students work on a common mathematical task (Jansen, 2012).

Conceptual Understanding. For the purposes of this paper, I define conceptual understanding as a student's ability to "[recognize] and [understand] core underlying [mathematical] ideas" (Burns et al., 2015) and to recognize how such ideas are interrelated (National Research Council, 2001). This stands in contrast to procedural skill, which is a student's ability to execute the steps needed to solve a problem (Rittle-Johnson, Siegler, & Alibali, 2001). I also explored how a student's ability to apply mathematical concepts to real-world situations is connected to conceptual understanding. Students with conceptual understanding are better able to apply mathematical knowledge learned at school to situations in everyday life than students who only have procedural skills (Kilpatrick, Swafford, & Findell, 2001). Further, the Common Core State Standards state that "mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace," thus emphasizing the importance of conceptual understanding (CCSS. Math. Practice.MP4, 2017).

Behavioral Engagement. I also sought to explore how small-group instruction might impact the behavioral engagement of students. Behavioral engagement is defined as students' active participation in learning activities (Wang, Berlin, & Berlin, 2014). Asking questions, sharing answers, and making related comments all may be indicators of behavioral engagement. Student engagement is essential, as it has been shown by research to be an indicator of academic achievement (Dotterer & Lowe, 2011).

Literature Review

Small-group Instruction in Mathematics. Past research has demonstrated that small-group instruction can be used to enhance student learning. For example, Kazemi and Stipek (2001) describe how small-group discussion in which teachers press students for justification of their mathematical ideas can help students move beyond their current level of understanding. Building on this research, Webb and colleagues (2009) compared the nature of elementary math teachers' interactions with students in small groups with their interactions during whole-class instruction. Results suggest that teacher probing of students' ideas in small groups may be more effective than probing during whole-class instruction, leading to higher instances of correct and complete mathematical explanations. Given this, small-group instruction served as the foundation for my data collection.

Strategies for Supporting English Learners and Former English Learners. Research has demonstrated that some strategies are more effective than others at supporting ELs. Specifically, Walqui (2006) identified the following relevant strategies: modeling, bridging (connecting new concepts to prior knowledge), and contextualizing (relating concepts to everyday situations and language). These strategies can be applied to further support ELs during instruction in flexible ability groups. In addition to supporting students currently classified as ELs, research suggests many students who have been reclassified English proficient (former ELs) still require language support in order to succeed with the demands of content area literacy (Haas, Huang, & Tran, 2014).

Offering Choice as Differentiated Instruction. Similarly, research has also revealed strategies for enhancing both behavioral engagement and conceptual understanding in students who are "advanced," or ahead of the majority of the class. Tomlinson (2005) argues teachers should accelerate the pace of instruction for more advanced learners and offer opportunities to make choices. When students are allowed to make decisions about the materials they use, the problems they solve, or the assignments they complete, they generally make choices that are more appropriate for their needs than what can be offered to the class as a whole; this, in turn, improves motivation and helps prevent disruptive behavior (Bluestein, 2008).

Math problems with Multiple Entry Points. Several math educators have advocated for Low-Floor-High-Ceiling (LFHC) tasks, also called Low-Threshold-High-Ceiling tasks, as a way of providing meaningful activities to different types of learners. LFHC tasks can be accessible to all students because they have multiple entry points; students can begin the problem at different levels. However, these problems also can be extended to higher levels depending on students' ability levels. The following problem is an example of a LFHC task, adapted from YouCubed (2016):

For each part of the problem, start with a square sheet of paper and make folds to construct a new shape. Then, explain how you know the shape you constructed has the specified area.

1. Construct a square with exactly ¼ the area of the original square. Explain how you know that this new square has ¼ of the area.
2. Construct a triangle with exactly ¼ the area of the original square. Explain how you know that this new triangle has ¼ of the area.
3. Construct a square (i.e. not a rectangle) with exactly ½ the area of the original square. Explain how you know that this new square has ½ of the area.

Such tasks allow students to work at their own pace, while also providing opportunities for challenge and critical thinking (Bernander & Metke, n.d.). LFHC tasks are designed to allow students to "show what they can do, not what they can't" (NRICH, 2011). Thus, these problems offer the potential for differentiating instruction while allowing all students to access grade-level concepts.

Methodology

Context and Participants. All research was conducted while the first author was the teacher of record in a self-contained 6th grade classroom. Of 28 students, 17 were ELs, and six had been reclassified fluent English proficient (R-FEP). All ELs were native Spanish speakers. School-wide, 77.5% of students qualified for Free-and-Reduced Lunch, an indicator of low socio-economic background. All classroom instruction and student discussions took place in English. Based on the previous year's standardized math scores, 71% of the class classified as "Standard Not Met," 29% as "Standard Nearly Met," and no students were classified as "Standard Met."

Table 1 displays background characteristics of the six focal students. Compared to their peers, students assigned to Intervention Group 1 were the most in need of support in regards to division and related word problems. In contrast, students in Intervention Group 2 had demonstrated an ability to work at a faster rate of instruction than the majority of their peers and had proven that they could successfully perform relevant skills independently.

Students were grouped fluidly based on their ability level of related concepts. When forming groups, I largely relied on student data collected during the lesson. For example, students answered questions on white boards during the lessons, participated in pair-shares and class discussions, and were encouraged to ask questions. Listening and recording students' responses offered data sources that helped me track how well certain students comprehended the concept at hand. Additionally, my own knowledge of student strengths and needs further helped me formulate groups.

Because of the fluid nature of the grouping process, the composition and size of the small groups were different for each lesson. For the purposes of this study, however, the six focal students remained in the same groups throughout all six rounds of data collection. It is important to note that small grouping is a strategy that I used frequently in my classroom across subjects, not just during data collection. As a result, the six focal students were assigned to different groups during other lessons. While some research has argued that ability level grouping can negatively impact students socially, I found that many students in my class *wanted* to be in the small group that received intervention instruction, and thus, more teacher attention. It should be noted that standardized test scores are included in Table 1 as a source of background information, not as a means of how students were assigned to groups.

Table 1: Focal Student Background Characteristics

Focal Students (*Names have been changed)	**Group**	**Common Core Standardized Math Assessment**	**California English Language Development Test (CELDT)**
Alex*	Intervention 1	Standard Not Met	Early Advanced (4)
Brandon	Intervention 1	Standard Not Met	R-FEP
Diana	Intervention 1	Standard Not Met	Intermediate (3)
David	Intervention 2	Standard Nearly Met	R-FEP
Megan	Intervention 2	Standard Nearly Met	R-FEP
Sammy	Intervention 2	Standard Nearly Met	R-FEP

The math content emphasized in interventions was guided by the district-adopted curriculum, GoMath! (Houghton Mifflin Harcourt).

Data Collection and Analysis. I position this study as "teacher action research" drawing on Cochran-Smith's and Lytle's (1993) definition, "systematic and intentional inquiry carried out by teachers" in their own classrooms for the purpose of taking action that has the potential to improve learning (p. 3). In this study, I analyzed the performance and growth of six focal students, conducting research in two phases. See Table 2 for a description of data collection across phases. Phase one consisted of four rounds of data collection focusing on the use of small-group differentiated instruction to meet the unique needs of each group. Phase two consisted of two rounds of data collection exploring the use of small-group LFHC tasks as another strategy for differentiating instruction. The basic format of instruction was the same for all rounds. Before I facilitated small-group instruction, I conducted a whole-group lesson in which I modeled a mathematical concept and corresponding skills. I subsequently divided students into small groups of 5-7 students based on current ability levels for the specific math concept. Behavioral engagement data was collected during both whole-group and small-group instruction.

Table 2: Summary of Data Collection

	Phase One				**Phase Two**	
	Round 1	**Round 2**	**Round 3**	**Round 4**	**Round 5**	**Round 6**
Conceptual Understanding Data	Pre-Interview, Quiz 1	Quiz 2	Quiz 3	Post-Interviews, Quiz 4	Pre-Interviews, Written free-response question, Quiz 5	Written free-response question, Quiz 6
Behavioral Engagement Data			Observ-ational field notes, Videotaped small-group instruction	Observ-ational field notes, Videotaped small-group instruction	Observ-ational field notes, Videotaped small-group instruction	Observ-ational field notes, Videotaped small-group instruction

Phase One: Differentiated Small-Group Instruction

Intervention Group 1. Because this group consisted of ELs and former ELs who struggled to understand word problems, I worked directly with the group using the strategy of contextualizing (Walqui, 2006). I led students in discussing and visually representing problems that required students to apply the mathematical concept taught during whole-group instruction.

For example, the focus of the lessons and activities in Round 4 was Order of Operations. During small-group instruction, students were asked to solve the following problem, $(\frac{1}{2} + \frac{3}{4}) \div 2$. Students and I collaboratively contextualized this problem by creating a 'real-world' scenario that described the problem and created a picture to represent the scenario. The students contextualized the fractions saying they represented "½ of a chocolate cake and ¾ of a cheesecake." They then created pictures to represent each mathematical operation.

Intervention Group 2. This group received an adapted assignment that required them to apply the same mathematical concept, but with larger numbers and multiple steps. I also encouraged students to show multiple ways of solving each problem to offer a greater challenge and opportunities for students to make connections between solution strategies

(National Research Council, 2001; Tomlinson, 2005). Students were allowed to choose how they visually represented each problem (Tomlinson, 2005). Students largely worked independently, however, when necessary, I provided students with 'hints' and guidance in which I referenced notes and anchor charts and emphasized keywords to help students understand the situation described in word problems as they worked.

Phase Two: Low-Floor-High-Ceiling Tasks

My second phase of data collection explored the use of two LFHC tasks with students working in flexible ability level groups for two rounds of data collection. All groups worked on the same LFHC task, however, as was the case in Phase One, I focused my data collection and analysis on the six focal students in Intervention Groups 1 and 2. I designed the LFHC tasks based on activities and information published by Cambridge University's NRICH (2011) and Stanford University's You-Cubed (2016). Both LFHC tasks were designed with the intention of being accessible to all students, yet open-ended enough that students could explore them at more advanced levels if appropriate (Cohen, 1999). Students were encouraged to focus on the exploration aspect of the tasks and to consider multiple solutions and approaches to the tasks.

Measurement of Conceptual Understanding. Students completed a three-question quiz at the end of each round of data in Phase One and Phase Two. Each quiz included three types of questions (a symbolic representation, a pictorial representation, and a word problem) because an ability to apply the same concept to different representations is an indicator of conceptual understanding (Panasuk, 2010). I scored each quiz using a modified version of the publicly available rubric for extended mathematical response items created by the Smarter Balanced Assessment Consortium (see Appendix A). I chose to use this rubric because it is used to measure students' mathematical understanding on the standardized assessments aligned with the Common Core Standards.

I conducted student interviews for each phase of the project. Pre-Interviews served as baseline data for conceptual understanding. Interviews were semi-structured and meant to assess students' conceptual understanding. Phase One interviews focused on division, as division was a recurring concept that students dealt with over the entire course of Phase One. Specifically, the interview questions were meant to assess whether or not students were able to identify a connection between the concept of division and other mathematical concepts and skills. For example, one question asked, "When you are dividing, what skills do you use to help you?" Another question asked, "When you think about division, what other concepts in math might division be related or connected to?" I audiotaped, transcribed, and then coded the interviews based on four coding categories created through a deductive process (see Appendix B).

During Phase Two, I collected pre-interview data on students' ability to apply a given mathematical concept to real-world situations, another indicator of conceptual understanding (Kilpatrick et al., 2001). Again, interview questions emphasized conceptual understanding, with questions such as, "In your own words, what is area?" and "When might we use area in our everyday lives?" Similar to the Phase One interviews, I audiotaped and transcribed the interviews, and then subsequently coded them based on three categories.

In Phase Two, I also asked students to create and solve their own "real-world word problem" via a written response question at the end of each round. I coded student responses using the same coding categories created deductively for the Phase Two Pre-Interview data (see Appendix C).

Measurement of Behavioral Engagement. I captured features of students' behavioral engagement by tallying the instances in which the six focal students showed one of the following indicators of behavioral engagement during whole-class instruction: answering a question (voluntary or involuntary), asking a math related question, or sharing a math related comment/answer. I then analyzed the video recordings of small-group instruction for Intervention Groups 1 and 2, counting the number of times each of the six focal students showed one of the aforementioned indicators of behavioral engagement.

I used the behavioral engagement data collected during whole-group instruction as baseline data. I used this data to then project the number of behavioral engagement indicators that students would demonstrate during small-group instruction for each round. To create the projections, I first determined how many times more students the whole-class instruction had as compared to each small group. I then multiplied this number by the number of behavioral indicators shown during whole-class instruction for each focal student to create a projection. Finally, I compared my projected data for each focal student to the data I gathered during small-group instruction.

Results and Discussion

Developing Connections Between Concepts. The culmination of data suggests students' conceptual understanding and behavioral engagement increased over the course of the study. During the Phase One pre-interviews, only three students identified a connection between division and another mathematical concept or skill (see Appendix B). These data suggest students were still developing an ability to identify how division is interrelated to other math concepts (National Research Council, 2001).

Post-interview data suggest all six focal students deepened their conceptual understanding of division in each of the four coding categories. In fact, all six focal students were able to identify that multiplication is a necessary skill for performing division. For example, Alex stated, “To divide you need to practice your multiplication.”

Additionally, two students recognized that multiplication is the inverse operation of division, while four students articulated that multiplication is related to division. In Sammy’s words, “Because like it’s like [multiplication and division] are basically the opposite of each other.” While no students were able to identify that division can involve other mathematical operations in the pre-interview, three students were able to do so in the post-interview.

Based on these data, it appears that students in both Intervention Groups 1 and 2 deepened their conceptual understanding of division in Phase One. While it is difficult to draw comparative conclusions about students in Groups 1 and 2 due to the small sample size, the data suggest students in Intervention Group 2 had a somewhat stronger conceptual understanding of division than students in Intervention Group 1. Even so, post-interview data encouragingly suggest students were increasingly identifying connections across mathematical concepts and, thus, had expanded their conceptual mathematical understanding (National Research Council, 2001).

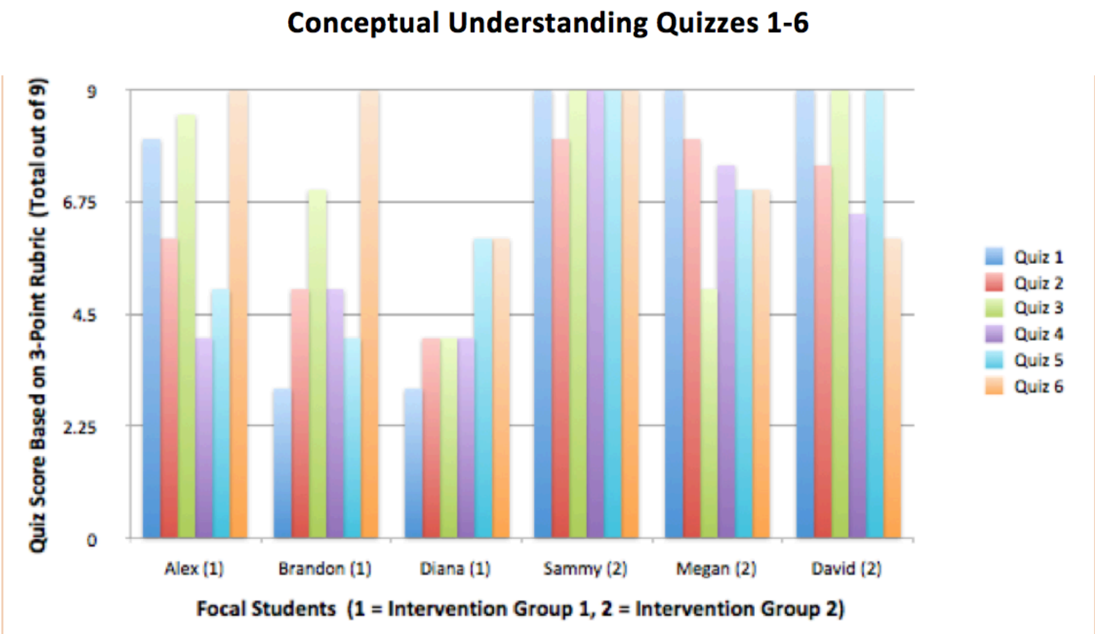
Connecting Concepts to Real-world Applications. In Phase Two, I assessed conceptual understanding based on students’ ability to apply mathematical concepts to real-world situations (Kilpatrick et al., 2001). Pre-interview data suggested students were not fully able to apply a given mathematical concept, in this case area, to a real-world situation (see Appendix C). When asked to describe a situation in which area would be used during pre-interviews, only one student was able to describe a specific situation, and only three students were able to identify and describe the relevant mathematical operation needed to solve an area problem. For example, Sammy described, “Like isn’t it base and height? And they’re kind of like put together and you know…you multiply them to be able to find the area.”

After engaging in LFHC tasks, students were asked to write and solve their own “real-world word problem” about area or volume. Data across the two rounds with different LFHC tasks revealed that more students were able to apply mathematical concepts when completing the written response questions than they were during the pre-interview. All six students were able to describe a specific real-world example involving the given mathematical concept (area) in Round 5, and four students were able to do so in Round 6, which focused on the concept of volume. The two students who were not able to create a real-world example of volume in Round 6 instead wrote a word problem about area, suggesting that these students need more opportunities to explore the differences and connections between area and volume.

Overall, data suggest students did develop a greater ability to apply mathematical concepts to real-world situations after engaging in LFHC tasks. For example, one student stated: "I have a tissue box and I want to see how much tissues fit into it. What is the volume if the height is 8 in, the width is 6in, and the length is 10 in?" Students were more frequently able to describe specific situations involving a given math concept in Rounds 5 and 6 than they were during the pre-interview. This indicates the LFHC may have helped students deepen their conceptual understanding.

Applying Concepts to Different Types of Questions. Quiz data revealed several interesting trends. In comparing average scores for each type of question across quizzes, we found no meaningful differences; the type of question that students struggled with most on each of the six quizzes varied by round. While students did not demonstrate complete conceptual understanding of the concepts that were assessed by each quiz, the fact that they were able to at least partially answer three different representations of the same concept suggests students were, indeed, beginning to develop conceptual understanding (Panasuk, 2010).

Figure 1. Graph of quiz scores for focal students. This graph illustrates that students were able to apply mathematical concepts to different types of questions.

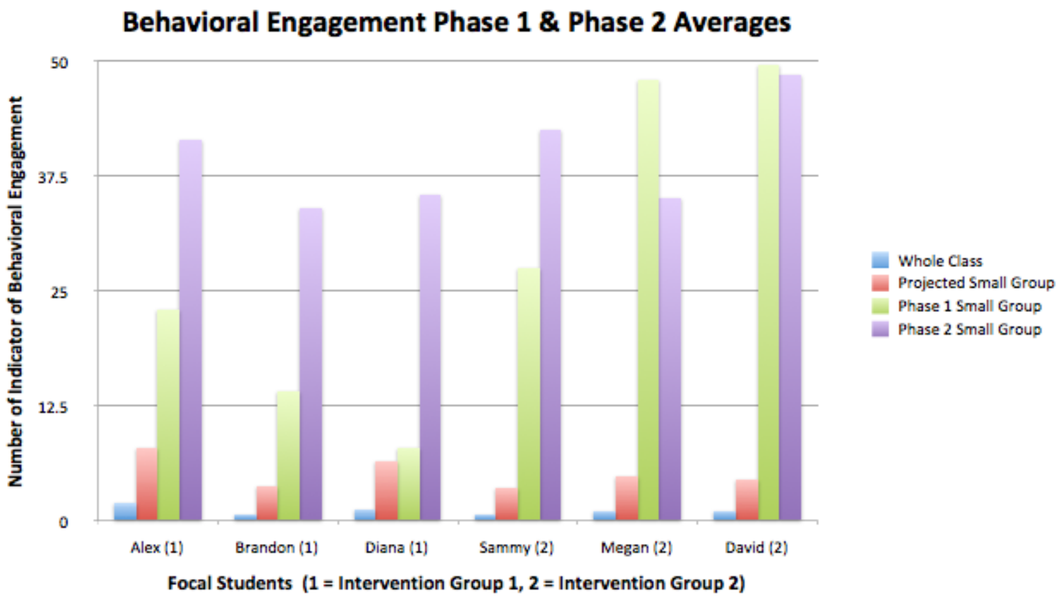


The overall quiz scores for each focal student (see Figure 1) show varied results. The results for Sammy (Intervention Group 2) show signs of the ceiling effect; despite the fact that instruction and quizzes become progressively more challenging over the course of the data collection, the quizzes were likely not challenging enough for Sammy. Even so, the combination of Sammy's quiz scores, interview data, and written responses reveal that Sammy had likely developed conceptual mathematical understanding. This appears to also be at least partially true for the other two students in Intervention Group 2, Megan and David; while Megan and David's scores fluctuated over the course of the four rounds, they ultimately demonstrated signs of conceptual understanding based on interview, written-response, and quiz data.

Students in Intervention 1 also showed signs of improvement in regards to conceptual understanding. All three students in Intervention Group 1 showed upward-trending quiz scores over the course of the six rounds. This was especially true for Brandon and Diana. Based solely on quiz data, it appears that students had not yet fully developed conceptual mathematical understanding. Considering interview and written-response data in conjunction with quiz scores, however, suggests that students were certainly in the process of developing conceptual understanding. Each student in Intervention Group 1 was able to recognize at least one connection between division and another mathematical concept, and each was able to identify at least one specific real-world example of a concept. In short, my quiz, interview, and written-response data suggest that small-group differentiated instruction and LFHC tasks do appear to help students develop conceptual understanding (Jansen, 2012).

Behavioral Engagement. Students in both Intervention Groups 1 and 2 clearly showed an increase in behavioral engagement during small-group instruction as compared to whole-group instruction (see Figure 2). This was true in both Phase One and Phase Two. I found no meaningful differences between the baseline data for Intervention Groups 1 and 2; the focal students in both groups showed an average of 1.1 behavioral engagement indicators during whole-class instruction.

Figure 2. Graph of behavioral engagement indicators for focal students. This graph illustrates that students in both groups were more behaviorally engaged during small-group instruction than they were during whole-group instruction.



All six focal students showed more indicators of behavioral engagement during small-group instruction as compared to whole-class instruction, as well as the small-group projected data. I projected that students would, on average, show 5.2 indicators of behavioral engagement during small-group instruction. In reality, students in Intervention Group 2 exhibited an average of 41.7 behavioral engagement indicators during Phase One, and 42 indicators during Phase Two. Students in Intervention Group 1 demonstrated an average of 15 indicators of behavioral engagement during Phase One, and 37 indicators in Phase Two.

These data, therefore, suggest that students in both small groups were more behaviorally engaged during small-group instruction than they were during whole-class instruction (Jansen, 2012). Even when the size differential between the whole-class and small-group instruction was controlled for, students still showed notably more signs of behavioral engagement during small-group instruction. Students in Intervention Group 2 exhibited almost an identical number of behavioral engagement indicators during Phase One, when students participated in small-group differentiated instruction, and during Phase Two, when students completed LFHC tasks. This suggests that both instructional strategies were an effective way to behaviorally engage higher performing students.

Students in Intervention Group 1, however, showed significantly more indicators of behavioral engagement during Phase Two (37 indicators) as compared to Phase One (15 indicators). This suggests that the use of LFHC tasks is a way to more equally engage students at both ends of the achievement spectrum (Cohen et al., 1999).

Implications and Conclusion

The culmination of data suggests that flexible small-group instruction can, in fact, be used to enhance both conceptual understanding and behavioral engagement for students at opposite ends of the achievement spectrum. Future research is needed to further investigate the use of flexible groups, however this research suggests that differentiated instruction and low-floor-high-ceiling tasks are both promising tools for targeting the unique needs of advanced and intervention small groups. Students in Groups 1 and 2 all showed signs of growth in conceptual understanding and behavioral engagement.

We believe that these instructional practices and findings may be generalizable to other classrooms with similar populations of students. Many teachers face the challenge of simultaneously catering instruction to students with different learning styles, needs, and strengths. This research suggests that flexible ability level grouping and LFHC tasks have the potential to differentiate instruction effectively.

Maya Angelou once said, "….in diversity there is beauty and there is strength." While most educators would agree with the wisdom behind this quote, we believe many would also add that effectively meeting students' diverse learning needs is one of the most challenging and worthy classroom goals. This inquiry project explored strategies of a first-year teacher that allowed her to meet her diverse students' needs and support *all* students in succeeding in the classroom, offering a glimpse into the potential of small-group instruction that encourages students to be involved in the learning process.

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Appendix A: Quiz Rubric

Adapted from Smarter Balanced Mathematics General Rubrics

Retrieved November 11, 2015 from https://www.smarterbalanced.org/wp-content/uploads/2015/08/Smarter-Balanced-Mathematics-General-Rubrics.docx.

0	**1: Partial Understanding**	**2: Reasonable Understanding**	**3: Full and Complete Understanding**
Student did not attempt problem/answer is not interpretable	Student's response contains some of the attributes of an appropriate response. However, the response shows evidence of insufficient mathematical knowledge, errors in fundamental mathematical procedures, and/or other omissions or irregularities.	Student addresses most of the task in a mathematically sound manner. The response contains sufficient evidence of the student's competence in problem solving, reasoning, and/or modeling, but not enough evidence to demonstrate a full understanding of the processes he or she applies to the specified task.	Student addresses the task in a mathematically sound manner. The response contains evidence of the student's competence in problem solving, reasoning, and/or modeling, and contains the correct final answer.

Appendix B: Phase One: Pre and Post Interview Data

Conceptual Understanding of Division

A = Advanced, I = Intervention

Code	**Student Example(s)**	**Focal Students** Round 1 Pre-Interview	 Round 4 Post-Interview
Student recognizes that multiplication is the inverse operation of division.	"Because dividing is like the opposite of multiplication…"	Sammy (A)	Sammy (A), Megan (A)
Student identifies that multiplication is a concept related to division.	"It's related to multiplying."	David (A)	Sammy (A), David (A), Alex (I), Diana (I)
Student identifies that multiplication is a necessary skill for performing division successfully.	"Like knowing your times tables." "To divide you need to practice your multiplication."	Sammy (A), David (A), Brandon (I)	Sammy (A), Megan (A), David (A), Alex (I), Brandon (I), Diana (I)
Student identifies that division can involve other mathematical operations.	"We use subtraction in division. And we have to use multiplication."		Megan (A), David (A), Alex (I)

Appendix C: Phase Two: Interview & Written Response Data

Conceptual Understanding of Area and Volume

A = Advanced, I = Intervention

Code	**Student Example(s)**	**Focal Students** Round 5 Pre-Interview	 Round 5 Written Response	 Round 6 Written Response
Student gives a vague example of a scenario in which area or volume applies.	"Say you're building a house. You need to find the area to be able to make it perfect."	Alex (I), Brandon (I), Diana (I), Sammy (A), Melany (A)		David (A)
Student gives an example of a specific scenario in which area or volume applies.	"When you're going to paint a room, you use area to know how much paint to use."	David (A)	Sammy (A), Megan (A), David (A), Alex (I), Brandon (I), Diana (I)	Sammy (A), Megan (A), Alex (I), Brandon (I)
Student correctly identifies and describes the operation used to solve the situation described in the scenario. OR Student correctly solves the situation described in the scenario.	"You would multiply the base times the height." "You would use the A=bh formula."	Sammy (A), David (A), Brandon (I)	Sammy (A), Megan (A), David (A), Diana (I)	Sammy (A), Megan (A)

