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Journal of Teacher Action Research - Volume 7, Issue 2, 2021
practicalteacherresearch.com, ISSN # 2332-2233
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Journal of Teacher Action Research

Volume 7, Issue 2, 2021

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About the Journal

Founded in 2013, the Journal of Teacher Action Research (ISSN: 2332-2233) is a peer-reviewed online journal indexed with EBSCO that seeks practical research that can be implemented in Pre-Kindergarten through Post-Secondary classrooms. The primary function of this journal is to provide classroom teachers and researchers a means for sharing classroom practices.

The journal accepts articles for peer-review that describe classroom practice which positively impacts student learning. We define teacher action research as teachers (at all levels) studying their practice and/or their students' learning in a methodical way in order to inform classroom practice. Articles submitted to the journal should demonstrate an action research focus with intent to improve the author's practice.

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IMPROVING HIGH SCHOOL STUDENTS' UNDERSTANDING OF QUADRILATERALS BY USING PRE-CONSTRUCTED DIAGRAMS ON GEOGEBRA

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Abstract The purpose of this action research study was to discover a method for helping secondary students understand the properties and relationships of special quadrilaterals. Students are more likely to work from their concept image of a geometric shape rather than a memorized definition when asked to recall the shape's properties in order to solve given problems. Giving students opportunities to experience and explore examples of each shape helps to refine and build students' concept images of each special quadrilateral. In order to give students the opportunity to enhance their understanding of special quadrilaterals via exploration, we created a unit of study which employed pre-constructed diagrams created on Geogebra. We wished to discover what kind of effect working in a dynamic environment with scaffolded activities might have on students' understanding of special quadrilaterals. Comparing pre- and post-assessment data from three separate classes ($n = 67$ students) showed that students' overall understanding of special quadrilaterals increased as a result of the intervention activities. However, not all students reached the same level of understanding by the end of the unit. The exploration-based structure of the unit allowed individualized instruction. Thus, regardless of students' prior knowledge, the activities from the unit provided each individual with an opportunity to grow in their understanding of special quadrilaterals.

Keywords: teacher action research, high school geometry, Geogebra, special quadrilaterals

Introduction

Students usually rely on specific geometric examples to make sense of geometric theorems and definitions (Cunningham & Roberts, 2010; Fujita & Jones, 2007; Tall & Vinner, 1981). Unfortunately, when studying the properties of and relationships between special quadrilaterals, teachers often present students with definitions of each quadrilateral at the beginning of the unit without letting students explore the quadrilaterals themselves. Teachers then require students to verify the definitions and deduce any additional properties of each quadrilateral not mentioned in the definition (Salinas, Lynch-Davis, Mawhinney, & Crocker,

2014). Unfortunately, students are rarely successful in applying memorized definitions to solve problems involving quadrilaterals (Cunningham & Roberts, 2010). Therefore, it seems that teachers must adjust their approach for helping students gain a conceptual understanding of quadrilaterals.

One reason for teachers to avoid teaching quadrilaterals without student exploration is the students' level of geometric reasoning, commonly referred to as their van Hiele level (Van Hiele, 1959/2004). Van Hiele emphasized that students must progress linearly through the earlier stages of geometric understanding before being able to engage in formal deductive reasoning, which is the highest van Hiele level (Crowley, 1987; Lai & White, 2012). Therefore, in order for students to understand a particular special quadrilateral, they must explore concrete examples of the shape to recognize the properties of the figure (Battista, 2007). Through the process of viewing multiple examples of a particular special quadrilateral, students begin to form a working definition of the given geometric shape.

I experienced this first hand during my first few years of teaching. As a new teacher, I assumed that all of my high school geometry students had a basic understanding of each special quadrilateral. As a class, we wrote down the properties of each shape. I required students to memorize these properties and then immediately moved into trying to teach formal deductive proofs involving the properties of special quadrilaterals. At the conclusion of the unit, I always found myself frustrated. Most students could not see relationships between special quadrilaterals nor complete formal proofs with quadrilaterals on their own. After a few years of observing students' difficulties with quadrilaterals, it seemed that students never truly understood the properties of each special quadrilateral. This challenge motivated me to discover a different way to help students understand quadrilaterals and their properties. Therefore, the aim of this action research study is to employ a research-based teaching method to help students better understand quadrilaterals.

My research question is as follows:

In what ways, if at all, do students show growth in their geometric understanding of special quadrilaterals both during and after completing guided explorations of dynamic quadrilateral constructions?

Literature Review

van Hiele Levels of Understanding. In the 1950s, Pierre van Hiele noted that students learn geometry in a linear progression. According to van Hiele, a student must advance through five levels of thinking in order to fully understand a geometric system (Van Hiele, 1959/2004). These five levels are now known as van Hiele Levels of Geometric Thought. The descriptions of each van Hiele level stay relatively consistent throughout most research. Table 1 summarizes the descriptions that have been commonly used amongst researchers (Battista, 2007; Crowley, 1987; Fujita & Jones, 2007; Gutierrez, Jaime, & Fortuny, 1991; Lai & White, 2012).

Table 1: Descriptions of van Hiele Levels

VAN HIELE LEVEL	DESCRIPTION
PRE-RECOGNITION (LEVEL 0)	Students are unable to identify many common shapes.
VISUAL (LEVEL 1)	Students are able to identify shapes according to their appearance.
DESCRIPTIVE/ANALYTIC (LEVEL 2)	Students are able to characterize shapes by their properties.
ORDER/RELATIONAL (LEVEL 3)	Students are able to form definitions and establish relationships between shapes.
FORMAL DEDUCTION (LEVEL 4)	Students are able to develop and apply theorems within an axiomatic system.
RIGOR (LEVEL 5)	Students are able to apply geometric concepts to various mathematical systems.

As can be seen in Table 1, student-understanding of a geometric topic starts with informal conceptualization based on previous knowledge and experiences. As students are given opportunities to examine geometric concepts, their understanding ultimately progresses toward a formal property-based system (Battista, 2007). Van Hiele stressed the importance of teachers matching instructional design to meet the van Hiele level(s) of their students (van Hiele, 1959/2004). For example, students at a van Hiele Level 1 are not ready to use a book definition of parallelogram to deduce properties of a parallelogram. Rather, it may be more successful for these students to view multiple examples of parallelograms with the goal of determining what all of the examples have in common (Lai & White, 2012). Teachers must help students construct knowledge that will allow them to progress toward being able to reason deductively (Jones, 2000). Students cannot be expected to prove geometric properties if they cannot first identify the properties.

Teachers must be aware of their students' geometric thinking levels in order to design proper instruction. Traditionally, van Hiele levels have been measured by the number of correct answers on a multiple-choice test (Gutierrez, Jaime, & Fortuny, 1991; Hollebrands, 2007; Kutluca, 2013; Usiskin, 1982; Wang & Kinzel, 2014). However, not all researchers consider van Hiele levels to be discrete (Battista, 2007; Gutierrez, Jaime, & Fortuny, 1991). For example, when a student transitions from one level to the next, she may demonstrate thinking from both van Hiele levels. Because of the fluid nature of van Hiele levels, analyzing students' responses to open-ended questions may be more effective in determining students' van Hiele levels.

The average high school geometry student is at a van Hiele Level 2 (Jones, 2000). At Level 2, students can identify properties of shapes but do so on the basis of visual examples. Therefore, students need multiple representations of geometric objects in order to increase their van Hiele level (Crowley, 1987; Lai & White, 2012). As students become more familiar with the properties of individual shapes, they will begin to notice relationships amongst different shapes. Students who identify connections between objects are at a van Hiele Level 3. For example, a student at Level 3 can deduce that a rectangle can be called a parallelogram due to having all of the properties of a parallelogram (Fujita & Jones, 2007).

Students' Understanding of Quadrilaterals. This study is focused on teaching special quadrilaterals to high school geometry students. There exists a hierarchy of related geometric characteristics between certain quadrilaterals that allows students to explore conjectures to identify the hierarchical relationships (Salinas, Lynch-Davis, Mawhinney, & Crocker, 2014). Before entering geometry, many high school students possess prior knowledge of certain quadrilaterals, which comes from visual examples and/or previous encounters with the shape, commonly referred to as a *concept image* (Cunningham & Roberts, 2010; Fujita & Jones, 2007; Tall & Vinner, 1981). The concept image differs from a formal definition (also known as concept definition) of the shape and serves as the foundation for a students' mathematical engagement with the object.

Because students are more likely to work from their concept image of a geometric object, teachers should not ask students to memorize the formal definition of each special quadrilateral in order to derive the properties of each shape. Teachers can facilitate revisions of students' concept images by having students explore examples and non-examples of the object in order to achieve *concept attainment*, the authentic formation of a correct description of the concept being explored (Cunningham & Roberts, 2010; Salinas, Lynch-Davis, Mawhinney, & Crocker, 2014). Students must reach concept attainment for each type of special quadrilateral before they can identify hierarchical relationships (Salinas, Lynch-Davis, Mawhinney, & Crocker, 2014).

In order for students to notice hierarchical relationships, they need to compare and contrast the properties of each specific type of quadrilateral (Salinas, Lynch-Davis, Mawhinney, & Crocker, 2014). It is critical for students to notice the properties that remain consistent throughout most types of quadrilaterals. For example, parallelograms, rhombi, rectangles, and squares all have two pairs of parallel opposite sides. However, differences between

quadrilaterals occur when constraints are made to a set of properties in order to create a new type of quadrilateral (DeVilliers, 1994). For example, in order to create a rhombus, the additional constraint of having four congruent sides must be added to the set of properties for a parallelogram. Hierarchical definitions allow certain types of quadrilaterals to fall into multiple, overlapping subsets (DeVilliers, 1994). Hierarchical definitions prove to be more advantageous for learners once they progress to a van Hiele Level 4 because it reduces the amount of work to prove quadrilateral properties due to certain quadrilaterals being subsets of others.

Using DGS as an Instructional Tool. Dynamic geometry software (DGS) allows users to construct geometric objects (e.g. points and lines) as well as manipulate the objects to view geometric relationships between various parts of the construction (Erez & Yerushalmy, 2006; Kutluca, 2013; Poon & Wong, 2017). There are many brands of DGS available to teachers (Kurtz, Middleton, & Yanik, 2005). DGS supports inquiry-based learning by providing students multiple examples of the same constructed figure via dragging vertices of the figure while maintaining the constructed properties of the figure (Hollebrands, 2007; Obara & Jiang, 2009; Poon & Wong, 2017). Geometric exploration helps students at lower van Hiele levels progress toward deductive reasoning (Crowley, 1987).

Students must use their prior knowledge to construct figures with DGS (Battista, 2007). However, students at lower van Hiele levels will not know the essential properties required to accurately construct each type of quadrilateral. One way to address this challenge is to provide students preconstructed diagrams, which allow them to view, measure, and manipulate the components of the preconstructed geometric object (Battista, 2007; Poon & Wong, 2017). Activities involving pre-constructed diagrams ask students to make conjectures about properties of the figure and then to confirm their conjectures (Erez & Yerushalmy, 2006; Hollebrands, 2007). Through this exploration process via DGS, students gain the conceptual knowledge that may not have been accessible via lecture and memorization of definitions (Obara & Jiang, 2009). Activities must guide students to the desired outcome of the exploration (Jones, 2000; Kutluca, 2013). This scaffolding will provide students with an opportunity to develop and/or refine their concept image of the geometric object being studied.

Teachers must also be aware of potential student miscues. As students manipulate a pre-constructed object on DGS, they must realize that a pre-constructed diagram will represent a given geometric shape regardless of how it is manipulated (Battista, 2007; Erez & Yerushalmy, 2006). The geometric properties that remain unchanged via manipulation of the diagram are the critical attributes of the object (Erez & Yerushalmy, 2006; Hollebrands, 2007; Jones, 2000). If students can find the critical attributes of a pre-constructed diagram, they will be able to then advance in their van Hiele level.

Once students have identified critical attributes of quadrilaterals, DGS can guide students towards recognizing that certain quadrilaterals can be given multiple classifications (Erez & Yerushalmy, 2006). In this process, students identify that there are certain attributes that various quadrilaterals share. For example, students can manipulate a rhombus in DGS to look

like a square because both squares and rhombi have four congruent sides. It should be noted that rhombi are not required to have four right angles, which allows students to manipulate a rhombus in DGS to look like a square. DGS acts as a tool for students to make meaningful connections between the various quadrilaterals.

Not all students will be at the same van Hiele level in a class therefore DGS allows for individualized instruction (Fujita & Jones, 2007). Working on DGS with well-structured activities can allow students to work at their own pace and at their own level of geometric thinking. Students who need more support will have the ability to review multiple examples of the quadrilateral, discuss what they are finding with a peer, and receive support from the teacher. Extension problems can be written for students who work at a faster pace.

Methodology

Research Design. This action research study employed a mixed methods approach. In order to determine if dynamic geometry software (DGS) helped students' increase their van Hiele level of quadrilaterals, pre- and post-assessment data was compared. Various studies have measured the impact that DGS had on student-learning by using a similar approach (Johnson-Gentile, Clements, & Battista, 1994; Kutluca, 2013; Pitta-Pantazi & Christou, 2009). In particular, this study measured each student's van Hiele level of understanding quadrilaterals before and after the designed unit centered around student-explorations of pre-constructed diagrams on DGS. In addition to quantitatively comparing students pre and post van Hiele scores as well as pre and post free-write scores, we also examined students' definitions of the various quadrilaterals in order to describe the changes that occurred, if any.

Study Participants. This study was conducted at a rural high school in central Illinois. The students in this study were from three different sections of the regular (as opposed to accelerated) geometry course. Each section met every school day for a 50-minute class period. The textbook used for this course was *Core Connections Geometry* by the *CPM Educational Program*. All three sections were taught by the same teacher, who was also the main researcher in this study. A total of 63 students participated in the study: 28 females and 35 males. Of these 63 students, there were twelve 9th graders, forty-three 10th graders, seven 11th graders, and one 12th grader. Participants in this study were a diverse group of learners that represented a variety of grades and ability levels.

Instrumentation. Two assessments were given both before and at the conclusion of a six-day unit on quadrilaterals. The first assessment was an adapted version of Usiskin's (1982) "van Hiele Level of Geometric Understanding Test". Several studies (Hollebrands, 2007; Kutluca, 2013; Wang & Kinzel, 2014) have used Usiskin's (1982) "van Hiele Level of Geometric Understanding Test" to measure the van Hiele levels of participants. In order to examine students' understanding of special quadrilaterals, we used a subset of Usiskin's (1982) test questions that related to quadrilaterals. The Usiskin (1982) test provides an overall assessment of students' van Hiele levels.

The second assessment was a free-write about each special quadrilateral studied during the unit. Because researchers have found that a student's van Hiele level may differ from topic to topic (Gutierrez, Jaime, & Fortuny, 1991), we asked students to share their concept image for each special quadrilateral in order to detect subtle changes in students' van Hiele levels. Each free-write response was given a van Hiele level rating based on a rubric adapted from Gutierrez, Jaime, and Fortuny (1991). Because students studied seven different shapes, they received seven separate van Hiele level scores on the free-write.

Procedures. This study took place during a six-day unit on quadrilaterals (namely trapezoids, parallelograms, rhombi, rectangles, squares, kites, and general quadrilaterals). We decided that six days was sufficient for students to thoroughly examine the different quadrilaterals. Moreover, given the curriculum requirements placed on us by the school district, six days was the largest number of days we could afford to dedicate to the unit on quadrilaterals. Before the unit began, students individually completed the multiple choice and free-write assessments. Students who were classified at the same van Hiele level were then partnered to work together. Placing students in homogenous pairs eliminated some variability when trying to determine the impact that DGS had on students' van Hiele levels. Pairing students also addressed the limited number of computers available for student use.

After receiving instruction about the goals of the unit, students worked in pairs through each guided activity. Each pair of students worked for roughly forty-five minutes each day on the guided activities. Every activity required students to manipulate the pre-constructed diagrams using DGS on the computer (see Figure 1) in order to make observations about the shape with their partner. For example, students moved vertex B to vary the appearance of the rhombus to determine which geometric properties remained constant.

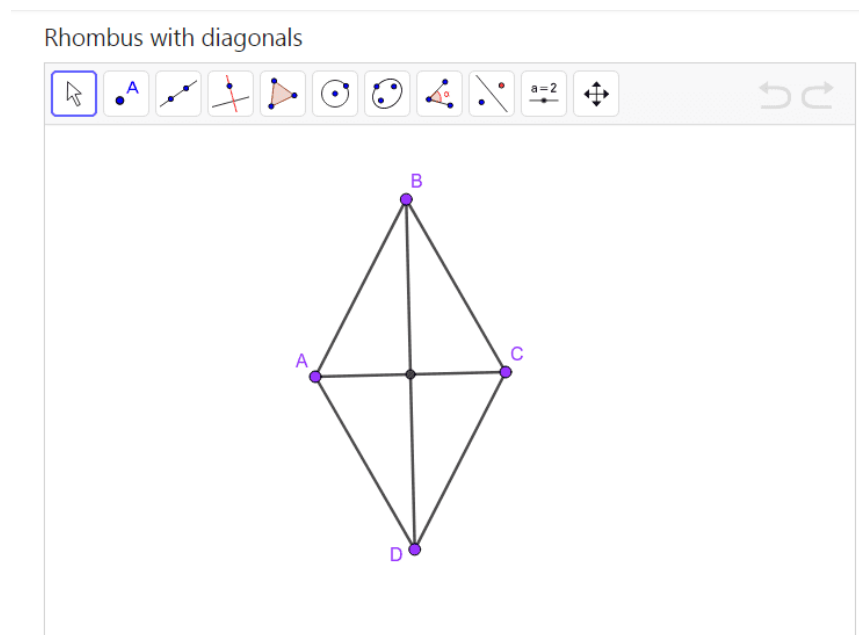


Figure 1: Sample pre-constructed rhombus on DGS used for the activities

Each activity had extension questions for students who completed their work early. The extension questions focused on trapezoids and kites, figures which do not receive much attention in textbooks. At the end of Activity 2, students produced a definition for each type of quadrilateral. Students' responses provided a formative assessment that helped the teacher measure progress mid-way through the study.

On the final day(s) of the unit, students shared their definitions of each quadrilateral with the class. The class then worked to synthesize their classmates' definitions in order to establish a class definition of each quadrilateral. Students also proposed hierarchical relationships between quadrilaterals. On the final day of the study, students took the two post-assessments.

Data Analysis

van Hiele Level Test. The multiple choice pre-/post-assessments were scored using the method found in Usiskin (1982) in order to assign each student a van Hiele level (see Figure 2). We then compared the mean scores of the pre and posttest using a one-tailed paired t-test (Cheraq & Shahvarani, 2017; Fields, 2005; Hsu & Lachenbruch, 2005). Our hypotheses are as follows:

$H_0: \mu_d = 0.$

$H_A: \mu_d \neq 0$

We chose to use an α -level of 0.10 because we were implementing the activities described above for the first time with a relatively small number of students and felt that ($p < 0.10$) would give us a sense of the potential of the activities for improving students van Hiele levels (Schumm, Pratt, Hartenstein, Jenkins, & Johnson, 2013; Taylor, 2020).

Scoring the Test (Usiskin, 1982):

- Each group of 5 consecutive questions represents a van Hiele level.
- If a student answers 3 or more questions from the group of 5 correctly, the student is considered to have achieved the corresponding van Hiele level.
- A student cannot "skip" a van Hiele level. Therefore, the first time a student misses 3 or more questions from a group of 5 questions, their van Hiele level has been set.

Figure 2: Scoring procedures for multiple choice pre-/post-assessments

Free-write Assessment. Each of the seven responses on a student's free-write assessment was assigned a van Hiele level based on the adapted rubric from Gutierrez, Jaime, and Fortuny (1991) (See Figure 3). Scoring was completed separately by two different researchers in order to minimize any bias and assure validity. We analyzed the data in two ways. First, we used a one-tailed paired t-test to compare the pre- and post-scores for each quadrilateral. We set our

alpha at 0.10 and our null hypothesis was that for each quadrilateral, the means for the pre- and post-free-write would be the same.

Next, we used the constant comparative method (Glaser & Strauss, 1967) to identify subtle patterns in the changes between pre- and post-free-write assessments. We hoped to be able to describe patterns of change (or lack of change) for the various quadrilaterals given the complex nature of students' transitions between different van Hiele levels (Battista, 2007; Gutierrez, Jaime, & Fortuny, 1991).

Level 0 (pre-recognition)	-Student leaves description blank -Nothing in the description is correct
Level 1 (recognition)	-Student describes the visual appearance of the shape ("It looks like") -Student only draws an accurate picture of the shape (without geometric markings representing properties) -Student's entire written description includes only irrelevant geometric attributes
Level 2 (Descriptive)	-Student uses correct geometric properties to describe the shape (although some properties may be incorrect or left out) -Student describes the shape in isolation of the other special quadrilaterals
Level 3 (Relational)	-Student uses correct inclusive classification or explains how a given shape is related to the other from the unit -Student mentions all of the necessary properties of the shape (and all additional properties are accurate with no mistakes)

Figure 3: Rubric for scoring the free-write pre-/post-assessments

Figure 4 shows examples of student responses to the term *parallelogram* along with their van Hiele level rating. We transcribed students' answers to the right for the reader to be able to understand students' writing. No details in Student A's response (Level 0) were correct. Student B's response (Level 1) focused on the visual appearance of a stereotypical parallelogram as well as provided a basic diagram of the shape. Although Student C (Level 2) and D's (Level 3) responses look very similar due to each listing several properties of parallelograms, Student D's response also included relationships parallelograms have with other special quadrilaterals.

Student A

van Hiele Level 0

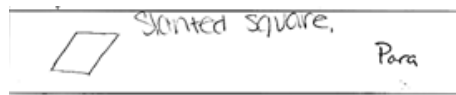
have 8 side but at different lengths

Parallelogram

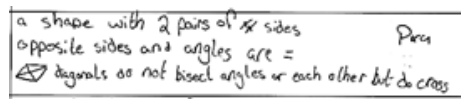
have 8 side but at different lengths

Student B

van Hiele Level 1

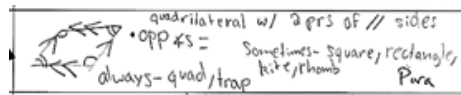


Slanted square

Student C*van Hiele Level 2*A shape with 2 pairs of \parallel sides

opposite sides and angles are =

diagonals do not bisect angles or each other but do cross

Student D*van Hiele Level 3*quadrilateral w/ 2 prs of \parallel sidesopp \angle 's =sometimes – square, rectangle,
kite, rhombus always –
quad/trap*Figure 4: Sample student-responses for “parallelogram” representing each van Hiele Level***Results**

The students' average overall van Hiele level for quadrilaterals before the unit was ($M = 1.54$, $SD = 1.01$) and after the unit was ($M = 1.78$, $SD = 1.30$). The results of the paired t-test showed $t(62) = 1.56$, $p = 0.062$. We therefore rejected the null hypothesis that the pre and posttest means were the same. Therefore, it seems that the intervention given between the pre-test and post-test had an effect in helping students increase their overall van Hiele levels. Given the more general nature of the van Hiele level test, we decided to examine each quadrilateral for changes between the pre- and post-implementations of the free-write assessment.

In order to quantify the changes in van Hiele levels for each quadrilateral, we used a one-tailed paired t-test to see if there was a significant difference between the pre- and post-scores of students' definitions. Our null hypothesis was that there would be no difference between the pre- and post-scores. Table 2 contains the 63 students' average van Hiele levels for each quadrilateral as well as the p-value for the one-tailed paired t-test and the Cohen's D score to determine effect size (Fritz, Morris, & Richler, 2012). Again, we set our alpha level at 0.10.

*Table 2: Comparison of average score on free-write assessment***COMPARISON OF AVERAGE SCORE ON FREE WRITE PRE-TEST AND POST-TEST (N=63)**

SHAPE	Pre-Test	Post-Test	P-value	Cohen's D
SQUARE	1.98	2.41	$p < 0.0000$	$d = 1.38$
RECTANGLE	1.89	2.33	$p < 0.0000$	$d = 1.08$

QUADRILATERAL	1.60	2.37	$p < 0.0000$	$d = 1.02$
PARALLELOGRAM	1.56	2.21	$p < 0.0000$	$d = 1.12$
KITE	1.46	1.81	$p < 0.0000$	$d = 0.62$
TRAPEZOID	1.43	2.17	$p < 0.0000$	$d = 1.00$
RHOMBUS	1.22	2.22	$p < 0.0000$	$d = 1.47$

Given that the p-value for each shape was $p < 0.0000$, we can reject the null hypothesis that there was no significant difference in the scores. Further, as can be seen in Table 2, Cohen's effect size value for each shape suggests that there was a significant effect from the activities in the unit when comparing pre- to post-test performances on the free-write assessments. Note that a score equal to one indicates that the difference between the pre- and post-scores is one full standard deviation. This is considered a large effect size (McLeod, 2019).

Each student's pre/post free-write was then compared to determine how many students showed growth. Each student could show an increase in van Hiele levels for up to seven quadrilaterals. As noted in Table 3, 34.92% of the students showed an increase in score for three out of the seven quadrilaterals, which was the most frequent occurrence. Only two students (3.17%) increased their score on all seven shapes. However, 82.54% of the students increased their score for at least three out of the seven shapes after completing the exploration activities.

Table 3: Frequency of increase in score on free-write exam

FREQUENCY OF INCREASE IN SCORE WHEN COMPARING FREE-WRITE PRE-TEST AND POST-TEST (N=63)

NUMBER OF SHAPES THAT STUDENT INCREASED SCORE ON	Number of students showing this increase	Percentage of the sample studied
7 SHAPES	2	3.17%
6 SHAPES	6	9.52%
5 SHAPES	12	19.05%
4 SHAPES	10	15.87%
3 SHAPES	22	34.92%
2 SHAPES	8	12.70%
1 SHAPE	3	4.76%

0 SHAPES

0

0.00%

Although all students increased in at least one score on the free-write assessment, we were also interested in the cases when students' scores did not increase. In fact, out of the 441 quadrilateral definitions we examined (seven definitions per student times 63 participants), 44.67% of the responses were rated the same van Hiele level (See Table 4). Of the scores that did not change, 87.31% were scored at a van Hiele Level 2.

Table 4: Frequency of cases when a free-write response score was maintained

LEVEL OF UNDERSTANDING WHEN SCORE WAS MAINTAINED FROM PRE-TEST TO POST-TEST (N=197)

LEVEL OF UNDERSTANDING	Frequency	Percentage
LEVEL 0	3	1.52%
LEVEL 1	15	7.61%
LEVEL 2	172	87.31%
LEVEL 3	7	3.55%

Given the majority of unchanged van Hiele levels were at a Level 2, we were curious to see if there was potential growth within the van Hiele Level 2. Figure 5 below shows the Level 2 rating from our rubric for the free-write (Gutierrez, Jaime, & Fortuny, 1991).

Level 2 (Descriptive)	<ul style="list-style-type: none"> -Student uses correct geometric properties to describe the shape (although some properties may be incorrect or left out) -Student describes the shape in isolation of the other special quadrilaterals
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Figure 5: van Hiele level two (adapted from Gutierrez, Jaime, & Fortuny, 1991)

We used the constant comparative method (Glaser & Strauss, 1967) to classify the 172 cases of students who remained at van Hiele Level 2. We arrived at two categories: *shows growth within Level 2* and *does not show growth within Level 2*. Figure 6 provides how we categorized the 172 student responses. We found that 82 out of the 172 responses showed growth within a Level 2. Note that students who showed growth within a Level 2 added to their knowledge of the particular quadrilateral but were still unable to identify hierarchical relationships between different quadrilaterals, which would classify the response as a van Hiele level 3.

Rubric for Sorting Free-Write Responses that Maintained a Level 2 Score from Pre- to Post-Test

Category 1: Shows growth within Level 2	Category 2: Does not show growth within Level 2
<ul style="list-style-type: none"> -Student listed additional properties of the shape on the post-test -Student eliminated listing irrelevant properties of the shape on the post-tests -Students showed a shift in focus to relationships the shape had with other shapes, however, the explanations were incomplete 	<ul style="list-style-type: none"> -Student's pre- and post-test response were identical -Student may have added additional properties of the shape on the post-test, however, did not include the essential characteristic(s) that sets the shape apart from the other special quadrilaterals

Figure 6: Categorical classification for sorting Level 2 free-write responses

In order to give us a more complete description of student growth, we sought to describe patterns in the type of growth between the pre- and post-free-write assessment. We noticed that there were different types of growth occurring with our students. Each pre/post definition fell into one of four groups described below in Table 5.

Table 5: Categorization of how free-write definitions changed from pre- to post-assessment

TYPE OF GROWTH	DESCRIPTION OF GROWTH
HIGH LEVEL 1 TO LOW LEVEL 2	Student's pre definition focuses on the look of the figure (e.g. it looks like a diamond). Student's post definition shifts to listing a few characteristics of the figure (e.g. it has parallel sides).
LOW LEVEL 2 TO HIGH LEVEL 2	Student's pre definition lists a few characteristics of the figure (e.g. it has parallel sides). Student's post definition shifts to an extensive list of characteristics of the figure. The post definition does not mention hierarchical relationships with other quadrilaterals (e.g. a square is a rectangle with equal sides).
HIGH LEVEL 2 TO A LEVEL 3	The student's pre definition has an extensive list of characteristics of the quadrilateral without mentioning hierarchical relationships with other quadrilaterals. Student's post definition shifts to the hierarchical relationships with other quadrilaterals (e.g. a square is a rectangle with equal sides).
NO GROWTH SHOWN	Student's definition did not change.

As can be seen in Table 6, when assigning all 441 responses to a group, responses were somewhat evenly divided at around 25% representation for each group.


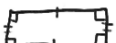
Table 6: Grouping free-write responses based on the growth shown

GROWTH SHOWN IN COMPARING STUDENT PRE-/POST-TEST FREE WRITE RESPONSES (N=441)



TYPE OF GROWTH	Frequency	Percentage
HIGH LEVEL 1 TO LOW LEVEL 2	109	24.72%
LOW LEVEL 2 TO HIGH LEVEL 2	82	18.59%
HIGH LEVEL 2 TO LOW LEVEL 3	126	28.57%
NO GROWTH SHOWN	124	28.12%

Figure 6 shows examples representing each type of growth described in the table above. Before the unit, Student E described a rectangle on the basis of the appearance (Level 1) but ended the unit describing a non-exhaustive list of its properties (low Level 2). Student F began the unit where Student E ended (low Level 2) but listed more properties of a rectangle by the end of the unit (high Level 2). Finally, Student G entered the unit understanding most properties of the rectangle (high Level 2) and as a result of the intervention ended the unit recognizing the hierarchical relationships the rectangle had with other special quadrilaterals (entering Level 3).

Student E: High level 1 to low Level 2

 <p>Like square except one longer side. Rect</p>	<p>2 pairs opposite sides equal with 90° angles.</p>  <p>Rect</p>	<p>Student F: Low Level 2</p>
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to High Level 2

 <p>A 4 sided shape with 2 pairs of equal sides. Rect.</p>	 <p>A quad. w/ 2 pairs of parallel sides. Opposite sides equal and parallel. And all 4s are 90°. Rect</p>
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Student G: High Level 2 to Low Level 3

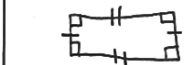

 <ul style="list-style-type: none"> • has 4 right 4s Rect • has 4 sides • has 2 pairs of equal sides • is a polygon • quadrilateral 	 <p>quadrilateral w/ 4 right 4s Rect</p> <p>has 2 pairs of // sides</p> <p>opp sides =</p> <p>always-quad, trap, rhomb</p> <p>sometimes- Rect, Square</p>
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Figure 7: Examples of student pre-/post-test responses falling into each growth group

Discussion

When reflecting on the findings, three pieces of data support the fact that students' knowledge of quadrilaterals increased due to participating in the guided exploration activities of the pre-constructed dynamic shapes on Geogebra. First, students showed a significant increase in van Hiele level scores on the multiple-choice assessment. Additionally, there was a significant difference between students' pre- and post-definitions for every quadrilateral. Finally, when looking at progress for individual students, all participants increased their knowledge for at least one quadrilateral from the unit: emphasizing that a majority of students increased their van Hiele level for at least three quadrilaterals.

As with any activity implemented for the first time, we found that several questions arose from the results. One trend we observed was students' van Hiele levels were not the same for each quadrilateral. On average, the participants had a stronger understanding of squares and rectangles as compared to all other special quadrilaterals. When the free-write pre-assessment was administered, many of the students skipped immediately to the questions that asked them to share their knowledge of the "square" and "rectangle" before attempting to write about any other shape. In many cases, these two written responses were scored higher than the responses for the five remaining shapes. However, after working through the unit and then taking the post-assessment, the students in the study showed less growth for squares and rectangles as compared to the other special quadrilaterals. We hypothesize that students may have lacked motivation in furthering their understanding of squares and rectangles because they believed their pre-existing concept image of these two shapes was sufficient.

Another trend we noticed was common misconceptions for certain quadrilaterals. The frequency of these misconceptions prompted researchers to wonder about how students experienced the quadrilaterals in previous lessons. First, many students claimed that a trapezoid must be isosceles, in other words, that the two non-parallel sides were congruent. Although the default setting for the pre-constructed dynamic trapezoid used in Geogebra was not isosceles, it seemed that students' pre-existing concept image of a trapezoid was so deeply rooted that students were unable to overcome this misconception.

Although students had difficulty with trapezoids, the intervention did help students correct some misconceptions. In particular, many students entered the unit thinking that rhombi and parallelograms have "slanted" sides whereas rectangles and squares have "horizontal" and "vertical" sides. Given that quadrilaterals are usually presented in textbooks with one side serving as the base, students' erroneous definitions seem reasonable. Students were able to overcome their misconceptions while using the tools on Geogebra because of the ability to orient the quadrilateral in any direction.

One finding that surprised us was that some students achieved a lower post van Hiele level rating. We wondered if the activity caused cognitive conflict within the student. In the pre-test, students may have provided memorized definitions that would receive a higher van Hiele

rating. Participating in student-led exploration that emphasized justifications of student claims may have caused students to question their memorized definition. As a result, the van Hiele level decreased because students were providing their own definition driven by a concept image instead of a memorized definition.

Conclusion

The motivation behind this research study was realizing that a high school geometry curriculum did not match students' van Hiele levels. Textbook activities attempted to promote formal deductive reasoning without providing foundational knowledge necessary to arrive at formal deductive reasoning. Results from this study seem to show that students can progress towards formal deductive reasoning via exploration and analysis of preconstructed quadrilaterals. This study also adds to the body of literature (Kutluca, 2013; Obara & Jiang, 2009) on the capabilities of DGS to help students transition through the stages of learning geometric content.

The findings also generated more questions as well as observations for how to improve the implementation of the geometry unit discussed in this paper. First, we observed that we need checkpoints in the activities in order to hold students responsible for their work as well as serve as formative feedback for the teacher. We also realized that we had unintentionally created a learning ceiling with the activities. The designed activities were not written to help students move beyond a van Hiele Level 3. Therefore, in the future, we hope to add activities that will allow the students at a van Hiele level 3 to move into a van Hiele Level 4 which focuses on deductive understanding of quadrilaterals.

Although we hoped for greater student growth, we feel that adjusting the activities and implementation of the unit could potentially bring about even greater improvement in high school students' understanding of quadrilaterals. It is our hope that teachers will try out these activities in their high school geometry classrooms and help improve this geometric unit that pairs inquiry-based learning with technology.

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