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MORE THAN WORDS: STRUGGLING READERS' COMPREHENSION OF WORD PROBLEMS

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Abstract Before they are able to solve mathematical word problems, students must be able to read and comprehend the problems. Although challenging for all students, struggling readers face additional cognitive demands when solving word problems that proficient readers do not. In this action research study, four focal students, including two English learners and two native English speakers, were given a multiplicative comparison problem and prompted to retell it in their own words, solve the problem using a selected strategy, and then retell the problem again. A retell rubric was used to analyze students' comprehension by measuring the completeness of the retell, while a drawing and writing rubric measured students' visual representations and metacognitive strategies involved in comprehension. Results suggest discussion provided opportunities for students to learn from one another and drawings especially helped English Learners' who communicated their understanding through symbols and visuals. Writing supported students' metacognitive skills leading to greater comprehension, but may be problematic for students at the beginning stages of English acquisition. It is imperative for teachers to provide opportunities for struggling readers to discuss, draw, and write about word problems to support their comprehension and to extend these skills to math in the real world.

Keywords: elementary math, word problems, reading comprehension, English learners

Introduction

This article describes an action research project conducted by a first-year teacher with the goal of increasing her understanding of how struggling readers comprehend mathematical word problems. Suggestions and implications for instruction are discussed.

Literature Review

Substantial research in the field of mathematics has been conducted on the use of discussions and thinks-alouds to comprehend mathematics. Researchers have found that when students are provided with opportunities to engage in meaningful mathematical dialogue, students' comprehension of a mathematical situation increases as so does their mathematical justifications (Bargh & Schul, 1980; Chi, 2000; Franke et al., 2009; King, 1992; Rogoff, 1991). Therefore, discussion can provide another way for students to interact with text, hopefully leading to more understanding of a presented mathematical situation in a given word problem.

Other researchers have found that the use of pictures and drawings supports students' comprehension and conceptual understanding of mathematics (Marino et al., 2010; O'Connell et al., 2005). Drawing a picture might help students connect what they think or say in a retell to a tangible representation in their solution process. It also allows students to communicate their comprehension of a word problem in multiple ways. Giving students opportunities to provide a mental model to a math problem allows them to internalize and contemplate a mathematical situation (Dexter & Hughes, 2011). Edens and Potter (2008) found that drawings can reduce the linguistic demand commonly found in word problems—an important finding when contemplating strategies for ELs who are struggling readers to use. Therefore, using drawings as a strategy to support students' comprehension of word problems is important to explore.

Writing has also been found to be a useful strategy as it supports students' metacognition (Artz & Armour-Thomas, 1992; Carr & Biddlecomb, 1998; Powell, 1997; Pugalee, 2001), an important thought process for students' comprehension. Although the test subjects from these research projects are much older in age (which seems questionable to use writing for fourth graders) Juliet Baxter and her colleagues (2005) found that 7th graders with writing and reading disabilities, operating at least two years below grade level, showed multiple instances of students' comprehension and conceptual understanding of problems when giving opportunities to journal their mathematical thinking. Accordingly, students who faced additional academic challenges, which affected their ability to write, still benefited from writing about their mathematical thinking. For this reason, providing opportunities for students to write about their thinking is important to investigate as writing could foster their comprehension.

Methodology

Context. This inquiry project was conducted in a fourth grade classroom at a public Title 1 school located in a suburban neighborhood known for gun and gang violence. About 86% of students at Applegate Elementary (pseudonym) receive free and reduced lunch, an indicator of high levels of poverty. Forty –two percent of students are ELs (English Learners) and Applegate is a program improvement school that has a strong focus on literacy.

The classroom consisted of 29 students, of which 14 were ELs. According to the California English Language Development Test (CELDT) that measures students' English proficiency on a scale of 1-5, the average EL level in the class was 3, representing an intermediate level English proficiency. Of the EL students in the classroom, 13 students' native language was Spanish and one students' native language was Hmong. Fifteen students were Hispanic, six white, five African American, and the remaining three were Pacific Islander, Alaska Native, or Hmong. Twenty-two students were reading below grade level, with 13 students reading one or more years behind grade-level norms.

For this inquiry project, four focal students were selected for in depth data analysis. These focal students were either reading at a first or second grade level, and were chosen because they represent the reading levels of a large portion of the class. Two of the focal students were ELs with CELDT levels 1 and 2 (beginning and early intermediate) and two students were EOs (students who speak English only). A combination of EL and EO students were chosen in order to explore how particular instructional strategies may support students with different linguistic needs. The four focal students are usually passive and their voices go unheard in group discussion. They are usually hesitant when solving word problems independently and wait for others to provide suggestions or answers. A primary goal of this project is to provide students with more opportunities to share their ideas and gain the confidence needed to attempt word problems. Prior to this study, the teacher primarily had students attempt word problems as a whole class with guiding questions to scaffold students thinking. However, the teacher desired a more student-centered approach by providing her students with strategies they could use to engage in mathematical thinking.

Purpose. In the field of mathematics, comprehension is crucial for students' success in word problems and in real world applications. Not only must students understand what a word problem is asking in real-life contexts, they must also be able to interact with the text of word problems to solve them. In a mathematical text, reading becomes even more difficult as text is not always read from left to right (depending if students need to also interpret and reference graphs/tables) and it is usually visually complex as there are callouts, sidebars of graphs, historical facts, and/or practice problems (Barton et al. 2002). These tasks become especially difficult for struggling readers as they face additional cognitive demands that proficient readers do not. When reading word problems, struggling readers are asked to simultaneously decode text, already an area of difficulty, while comprehending and relating these words to mathematics. Reading comprehension is strongly correlated with students' success on mathematical word problems (Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). Therefore, finding strategies that students can use to help lessen the cognitive demand of word problems is important to provide educational equity for our struggling readers in order for them to reach standards and use these skills in the real world.

Hegarty and colleagues (1995) define comprehension in mathematics on a tiered scale which involves (1) understanding the problem, (2) forming a plan to solve the problem and (3) carrying out the plan by solving it. For the purposes of this project, comprehension is defined as understanding the mathematical situation described in a word problem and being able to form a plan to solve it.

Multiplicative Comparison Problems are defined as "involving a comparison of two quantities in which one is described as a multiple of the other" (Carpenter, Fennema, Franke, Levi, & Empson, 2015, p. 66). These problems were emphasized in this inquiry project because they include language that can be particularly difficult for students (Stern, 1993), such as understanding the meaning of "twice as many," and they are prevalent in 4th grade math curricula and assessments.

As documented by state and district reading assessments, the current fourth grade class at Applegate struggles with reading and comprehending text. Moreover, at the beginning of the year, students completed a third grade math test containing word problems, many of them multiplicative comparison problems. The class average on this test was only 51%, indicating a need for additional support with this kind of problem. Multiplicative word problems have the added advantages of offering a window into students' understanding of the problem, because it is difficult to get a correct answer simply by "number grabbing" (Littlefield & Rieser, 1993)—where students pick the numbers seen in a word problem and randomly chose an operation without fully understanding the mathematical situation described.

The following research questions guided the design of this inquiry project:

1.) What strategies can struggling readers use to better comprehend multiplicative comparison word problems?

2.) Do ELs and EOs comprehend word problems differently, and if so, how?

3.) What parts of word problems are students struggling with?

Data Collection and Analysis. The effectiveness of retells to monitor and aid comprehension is well known throughout the literacy research community (Brown & Cambourne, 1987; Hoyt, 1999; Mowbray, 2010). Therefore, for each of the three rounds of data collection, students were prompted to (1) retell a presented word problem in their own words, (2) solve the problem using one of the three strategies, and (3) retell the problem again. The teacher conducted all three rounds. Students' retells were evaluated with a Retell Rubric (Ambrose & Molina, 2014), which unpacks word problems into their component elements, indicating the parts of the problem students understood or attended to, and the parts they did not. The elements of each word problem that were analyzed are the numbers, the units, the mathematical relationship, and the question. For each element, students who correctly retold that part of the problem received a score of 2, students who retold an element differently from how it was stated in the problem received a score of 1, and students who omitted an element completely received a score of 0 (Appendix A).

For Round 1, students were given a handout of the following problem: *Thomas built a fence that was 12 times as long as Terry's. Terry built a fence that was 4 feet long. How long was Thomas's fence?* They were prompted to retell the problem in their own words after rereading the problem as many times as they needed. For each retell, students worked with me one-on-one so that their peers did not influence their responses. Then, in partners (ELs and EOs were partnered together), students were prompted to discuss the word problem to one another, noting similarities or differences in thinking about the problem (a classroom norm). Here, students' conversations were audio-recorded and transcribed. Independently, students solved the problem on their handout. The teacher took field notes of students' explanations to their solutions. Lastly, students were again prompted to retell the word problem in their own words, rereading the problem as many times as they needed. The teacher reread the transcribed discussion and compared what students said in their explanations to the written work on the handout. This allowed themes to emerge about the match and mismatch between students' oral discourse and their written solution strategies.

For Round 2, students were given a handout of the following problem and prompted to retell it in their own words after rereading: *The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe?* The teacher then asked students to solve the problem by drawing a picture of it. Once they reached a solution, the teacher took field notes of students' explanations for their answers. Students were then prompted to retell the problem in their words again. To analyze students' drawings, an iterative process of creating codes was used by researching the necessary components in a drawing needed to comprehend a word problem (Dexter & Hughes, 2011; Edens & Potter, 2008) (Appendix B). Van de Walle's (2012) four-point rubric was modified by adding another category titled "Outstanding," indicating no errors in the drawing. This rubric, (Appendix C), allowed students' work to be analyzed against a set of desired learner responses and to place their understandings on a developmental continuum. Students were not expected to receive a perfect score on the drawing rubric as students' drawings were intended for them to make sense of the problem—not to present to an audience.

For round 3, students retold the following word problem in their own words: *Jill lived 5 times as many miles as Leo did from the ocean. Leo lived 20 miles from the ocean. How many miles did Jill live from the ocean?* The teacher then asked students to solve the problem by journaling, or writing their thoughts, about it. The teacher emphasized that their grammar and spelling was not important. After students journaled on their handout and arrived at a solution, students retold the problem again. Students' writing was analyzed in two different ways. First, using Ambrose and Molina's (2014) retell rubric was used to assess students' writing. This was analyzed to determine if students' retells were different when they were written from when they were spoken.

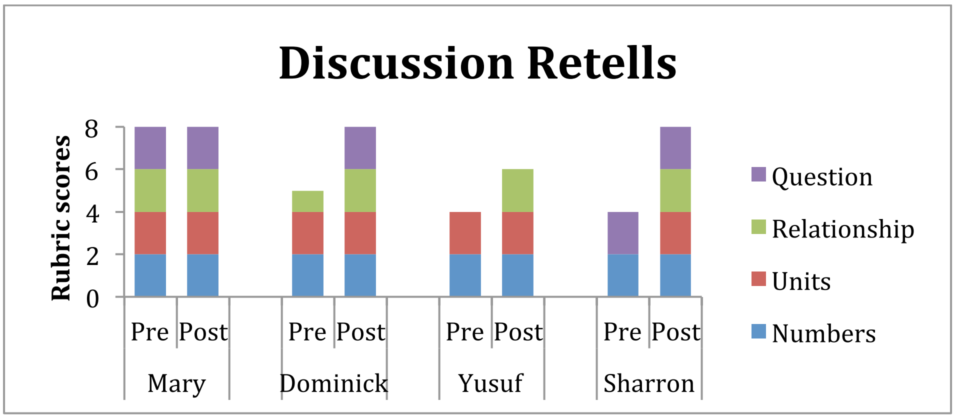
Second, students' writing was analyzed by going through a deductive process of creating a Metacognitive Strategies Rubric (Appendix D). Metacognitive strategies were analyzed because research indicates a strong correlation between metacognitive skills and students' comprehension (Lippmann & Linder, 2007). Drawing on the work of Tanner (2012), who suggests teachers support the development of metacognition by asking self-reflective questions for planning, monitoring, and evaluating, the teacher created a rubric to assess students' metacognition within their mathematical writing. Students received of score of 1 for "yes" and a score of 0 for "no." A total possible score was 3, although it was not expected students would receive a maximum score as they were asked to journal about the problem only for themselves and were not told their writing would be read by a distant audience.

Results and Discussion

Findings from Round 1 suggest the focal students' comprehension of word problems improved when they were given opportunities to discuss a problem with their peers. First grade level readers improved the most significantly, as seen in the figures below. For example, Sharron's ability to include elements of the original word problem during the second retelling increased by 50%, and Dominick's second retelling increased by 38%. Yusuf and Sharron were able to identify the relationship of the problem after their peer described it in the student discussion. These students who initially struggled with the concept learned from their peer once given opportunities to discuss. This finding is supported by other research studies as students learn from one another when engaging in student-talk (Franke et al., 2009).

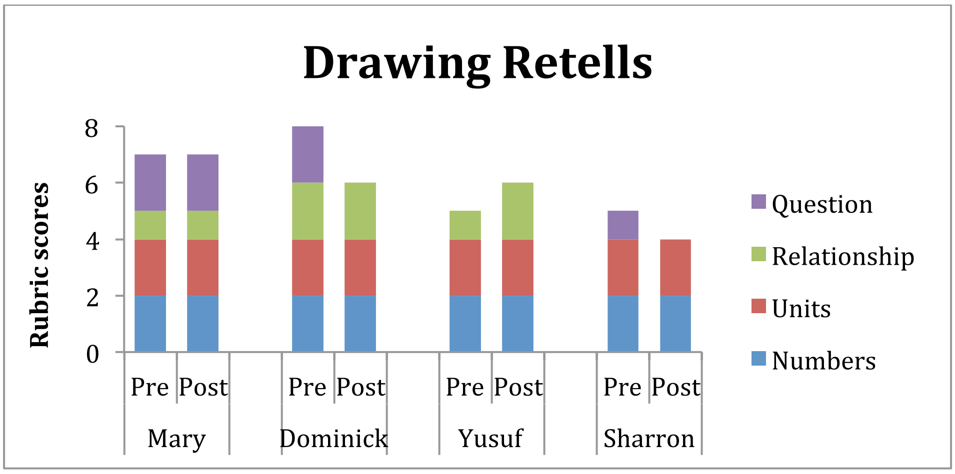
Although Mary was able to correctly restate the problem in her own words with 100% accuracy during the pre and post-discussion retellings, there was a misalignment between what she said and how she solved the problem. The other three students' retell matched how they solved the problem. This finding may indicate the limitations of using retells as a way to gauge whether students understand word problems. Perhaps as a compensation strategy for low reading ability, some students are able to memorize and restate a problem without really understanding the presented mathematical situation.

Figure 1: Graph of Pre- and Post- Discussion Retell Scores.



A similar phenomenon occurred in Round 2 when students' retells appear to worsen after the use of the drawing strategy, seen in Figure 2. However, three students solved the problem correctly and were able to explain their solutions, demonstrating their comprehension. The average score on the Drawing Rubric was 3.5 out of 5. No student included a representation of the unknown, which is mirrored by their retells. However, the drawing was intended to support students' comprehension, not to present a full picture to an audience. Drawings provided focal students with an alternative means to communicate their comprehension of the word problem. This was especially beneficial for the EL students (Dominick and Yusuf), perhaps due to a reduction in linguistic demand through drawing. Dominick represented the relationship between the animals' heights with an arrow and Yusuf drew a bar indicating differences in height. These samples are found in Appendix E. Students often pointed to their drawings to contextualize what they were explaining to the teacher in their planning process to solve the problem, a finding which is affirmed by prior studies of communication via drawing in mathematics (Dexter & Hughes, 2011; Edens & Potter, 2008). Although their retells were incomplete, students' comprehension of the word problem was represented through their drawings and explanations.

Figure 2: Graph of Pre- and Post- Drawing Retell Scores.



Findings from Round 3 suggest students benefited from writing about the math problem, displayed in Figure 3. Comparing the pre- and post-retells, we see that Yusuf and Sharron had similar difficulties retelling the relationship involved in this problem. This suggests that interpreting relationships in comparison word problems are equally difficult for EO and EL students. Both EL students, Yusuf and Dominick, also share similar improvements in their post-retell as both students were able to correctly identifying the numbers in the problem. This suggests that their writing might have helped them internalize the problem and associated numbers at deeper level than verbally speaking. This is supported by Baxter and colleagues' (2005) who found that students' comprehension of word problems was more evident in their writing than in their oral discourse. Sharron's and Yusuf's writing included more elements of the word problem than their previous retell, suggesting that writing gave students an opportunity to think about the problem more deeply. However, this pattern is reversed for Dominick, who included only the relationship in the problem in his writing and no other elements of the problem, as seen in Figure 4. This suggests that for CELDT level 1 students, writing may not be as effective a strategy to support students' thinking and comprehension of a problem. This finding is affirmed by researchers Edens & Potter (2008) who found that linguistic demands are decreased for EL students when pictures are utilized but are increased when writing tasks are required. It appears that writing may have encouraged students to formulate a plan to solve (Appendix F). However, when verbally prompted, students illustrated even more metacognitive skills. For example, Yusuf wrote, "you can do add up" but when asked by the teacher why would we add he stated, "Uh! No! You're suppose to times! Cause it says 5 times. So multiply!" This suggests that especially when paired with teacher questioning, writing can help students think about their justification for solving a problem in a particular way.

Figure 3: Graph of Pre- and Post- Writing Retell Scores.

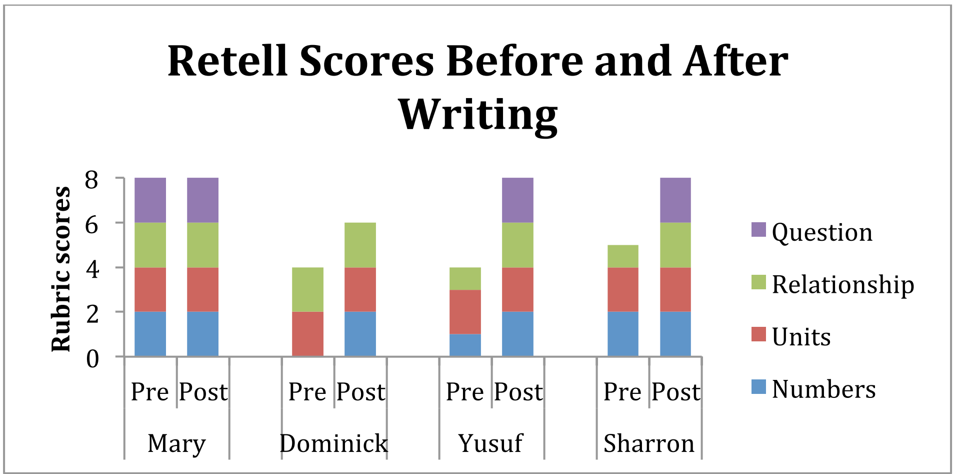
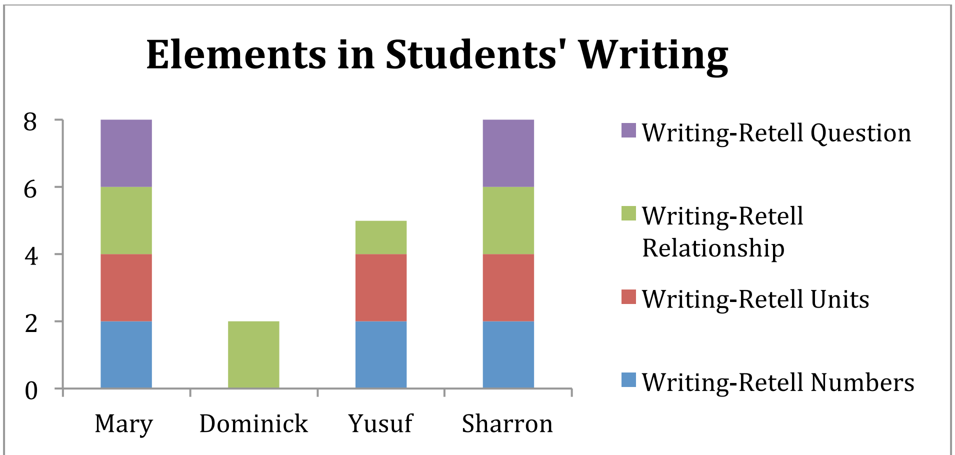


Figure 4: Graph of Elements Included in Students' Writing.



Conclusion and Implications

This action research project investigated struggling readers' comprehension of word problems utilizing three different strategies: discussing with a partner, drawing, and writing. Students with the lowest reading scores showed the most improvement after discussing the word problem with a partner. This suggests teachers should partner students of different reading levels together because struggling readers seem to benefit when partnered with higher-leveled reading partners. Drawing and writing about word problems can also be effective strategies for struggling readers, especially those who are English learners. Drawing was especially helpful for EL students perhaps because it offers an opportunity to communicate mathematical understanding through symbols and visual representations. Writing also seemed to support students' comprehension of word problems, although it created additional challenges for Dominick, an EL with beginning level English proficiency. Thus, teachers may want to be cautious when using writing-to-learn strategies with students who are at the beginning stages of English development. Beginning ELs may be better able to communicate their understanding through other modes, such as drawing, discussion, or using manipulatives.

Multiple and varied measures of assessment may help teachers gain a deeper understanding of their students' comprehension of word problems. Although they provide valuable information about students' understanding of a problem, using retells alone may be misleading. For example, Mary retold the problem with 100% accuracy but there was a misalignment between what she said and how she solved the problem. Furthermore, across all three rounds Mary's pre and post retell scores remained unchanged, although she did not always solve the problems correctly. This suggests retells alone are not enough to determine whether a student understood the problem. Multiple assessments, such a using a combination of retells, drawings, writing, and oral discussion, can offer a broader, perhaps more accurate view of students' comprehension of mathematical word problems. Future research can explore how different kinds of prompts for writing can elicit differences in students' mathematical writing, and whether it is more beneficial for students to draw a picture of the mathematical situation before or after discussing the problem with peers.

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Appendix A: Retell Rubric

Element	**Same** **(2)**	**Different** **(1)**	**Missing** **(0)**
Numbers	Contains both numbers from original problem.	Contains two numbers. At least one number is different than original.	At least one number in the problem is missing.
Units	Unit is identified in problem.	Unit is different from original problem.	Unit is missing.
Relationship	Relationship is consistent with word problem.	Relationship differs from what was described in word problem.	Relationship is missing.
Question	Contains a question consistent with word problems.	Contains a question with meaning different from original.	Question is missing.

Appendix B: Elements Within Drawing

Student	Kangaroo	Giraffe	Kangaroo's height	Giraffe's relationship	Unknown	Rubric Score
X	X	X	X	X	X	X

Appendix C: Drawing Rubric

No Attempt **0**	**Unsatisfactory** **1**	**Marginal** **2**	**Proficient** **3**	**Excellent** **4**	**Outstanding** **5**
The task is not completed. No elements in drawing are correctly indicated.	Fragments of accomplishment, but little to no success. Only one element in drawing is correctly indicated.	Part of task is accomplished, but lack of evidence of understanding. Two elements in drawing are correctly indicated.	Could work to full accomplishment with minimal feedback. Three elements are correctly indicated.	Drawing meets demands of task. May have minor errors. Four elements are correctly indicated.	Drawing meets demands of task with no errors and all five elements are correctly indicated.

Appendix D: Metacognitive Rubric

Student	**Planning** Does the student write about a kind of plan to solve the problem?	**Monitoring** Does the student write about any contemplation they are having about the problem?	**Evaluating** Does the student check to make sure an answer makes sense, or double check their thinking?
X	X	X	X

Appendix E: Drawings

Yusuf's Drawing **Dominick's Drawing**