Mathematics Word Problem Solving Through Collaborative Action Research

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Abstract: In this study, two researchers, a third-grade teacher and a professor of mathematics education, investigated the impact of explicit mathematical vocabulary instruction and substantive formative assessment feedback on third grade students’ abilities to solve word problems in mathematics. Authors worked together to observe, reflect, plan, and implement as part of a collaborative action research project. Once the first research cycle was completed, it was evaluated the interventions. Analysis of the qualitative data (interviews with students, observation and journal entries) and quantitative (surveys and exams) showed a significant improvement of students' word-problem solving abilities. Developing mathematical vocabulary enabled them to understand mathematical terms and requirements while providing feedback on problems assessment led to the improvement of the 'gap' in the process of problem solving.

Students must speak the language of mathematics to be successful in learning mathematics (Pimm, 1987; Moschkovich, 2012). Word problem solving in mathematics is an important aspect of learning mathematics and mathematical thinking. Unfortunately in everyday work, students exhibit difficulties solving word-problems, even when they may be skilled in performing other mathematics tasks. They easily execute basic mathematical operations such as addition, subtraction, multiplication, and division. These students ably identify units of measurement and perform calculation tasks with numbers and equations. However, when the operations are behind word problems, many students struggle to know what to do. In some instances students attempting to solve a word problem will be able to identify some elements of the problem but are unable to complete all of the required operations and will be unable to produce an acceptable answer.

As I watched the students in my third grade classroom struggle with solving word problems I began to ponder ways I could help students develop the abilities to solve word problems. I considered questions such as: what are the barriers that prevent students from solving word problems?; what are some instructional strategies that may be useful?; which activities can help students in this particular case?; which skills should students develop to be able to solve word problems? I shared these questions and concerns with a professor in the Faculty of Education at the University of Prishtina - Kosovo. She suggested that together we develop a collaborative action research project to answer these questions.

We chose to conduct action research rather than traditional research. Action research also known as practitioner research, is a systematic inquiry process. This type of research differs from traditional research in several ways. Traditional research is most often conducted by objective researchers that are unconnected to the research setting. These researchers define the study environments and therefore control the variables. Action research is undertaken by stakeholders to resolve specific and targeted problems (Springer, 2014). This type of research may be done by teachers for themselves (Mills, 2011).

Collaborative action research is the joint research between two or more teachers, or between university faculty and teachers. They collaborate and influence changes in the curricular approach and focus mainly on practical problems or individual teachers (Vula & Berdynaj, 2011). “Collaboration” is encouraged in teacher action research to bring co-researchers into an inter-subjective dialogue intended to open and refine different ways of knowing (Pine, 2009).
Peer-observation and professional buddying or mentoring systems are used widely in action research to promote collaboration. The prerequisite for a successful collaboration is mutual trust between professionals and common beliefs about what constitutes good teaching in their subject (Norton, 2009).

Collaboratively we performed a review of existing research studies and discovered that many mathematics teachers shared similar difficulties in helping students learn to solve word problems (NCTM, 2000; Sharma, 2001; Van De Walle, 2007; Burns, 2007). Researchers have studied the role of mathematical vocabulary and its impact in students’ achievements in mathematics and word problem solving (Amen, 2006; Blessman & Myszczak, 2001; Georgius, 2006; Brethouwer, 2008; Kranda, 2008; McConnell, 2008). It is not enough for students to learn mathematics only by solving tasks that require computations or memorizing concepts and operations. Students should be able to solve problems that encourage and develop thinking and logic skills. Problem solving is a skill that is required by life in general.

According to researchers, a particularly difficult part of solving word problems is the understanding of the problem, especially the words that are included in some problems. Not understanding certain words presents the first difficulties in word problem solving, causing misapplication of appropriate mathematical operations. Burns (2007) compares the learning of mathematics with learning of second language. Sharma (2001) also compares mathematics with language: “mathematics is a kind of language where communication takes place through the symbols, it has its letters, symbols, vocabulary and grammar” (p. 66). Students cannot be successful in mathematics if they do not know the meaning of essential vocabulary words. If students know the meaning of terms they can learn mathematical concepts and develop necessary skills in mathematics. This is true for all subjects, students must know the essential vocabulary of a subject to successfully learn the content.

Different research findings have shown that the development of mathematical vocabulary affects students’ abilities in mathematics. According to Blessman and Myszczak (2001), one of main causes of confusion in mathematics is vocabulary. Students need a stronger understanding of mathematical vocabulary to be successful in mathematics. Understanding of mathematical vocabulary influences the comprehension of lessons, tasks, various tests, especially in solving word problems, so a lack of understanding of mathematical terms affects capabilities to solve problems (Amen, 2006).

There seems to be a direct link between success in problem solving and vocabulary. A student’s ability to understand words in mathematics classes is related to its ability to solve word problems. Georgius (2006) found that students feel that the knowledge of the definitions of mathematical terms is significant and increases their achievements. Kranda (2008) conducted research about the relationship between students’ accurate understanding of mathematical vocabulary and their achievements, particularly focusing on understanding word problems and abilities to use appropriate mathematics language in word problem solving. The impact of vocabulary instruction for the understanding of mathematical concepts by student is researched by McConnell (2008). When students are directly instructed to use the language of mathematics, in many ways they develop better understanding of mathematical concepts and word problem solving becomes easier. Solomon (2009) showed that taking time to write words related to problems and discussing their meaning in the context of the problem, provides students more opportunities to know what to do with problems.
Student assessment is an integral and very important part in the learning and teaching process. Assessment is essential, not only to assess students’ achievements, but also for the purpose of improving their work. According to Burns (2007) assessment should focus on understanding students’ ideas, problem solving skills, and learning reactions. A good assessment can improve learning in many ways.

Feedback on assignments is also a valuable part of the learning process. Feedback can assist students in setting goals, taking responsibility for their learning, and becoming more independent in learning. It may also help students understand the characteristics of accurate and complete responses. To ensure high quality learning for all students, assessment and feedback should become a routine part of classroom activities.

This type of developmental assessment is called formative assessment. Formative assessment is a process of systematic observation that provides a better understanding of what students have learned and how to engage them deeply in the learning process. Observing and assessing students work in an on-going manner aims at improving the performance of students, motivating and orienting them to work in further activities. This process helps to “direct students in the learning process and enables them to acquire necessary skills that will be useful to achieve better results” (Murchan, Shiel, Vula, 2012, p.17). Formative feedback as part of learning assessment strategies provides information that students will use as a basis for improvement.

The research literature on improving students’ abilities to solve word problems in mathematics pointed us to two important classroom interventions: vocabulary and formative assessments. Our research questions were:

1. What is the impact of teaching mathematical vocabulary on students’ abilities to solve word problems?

2. What is the impact of formative feedback on the development of students’ abilities to solve word problems?

We conducted this research in a class of third grade students, in Yll Morina Elementary School in Gjakova, Kosovo. Yll Morina is a public school in an urban environment and has a total of 1229 students and 59 teachers. It is one of the most distinguished schools in Gjakova, recognized for the successes and academic achievements of students, as well as their participation and performance in competitions and extracurricular activities.

The total number of students in the third grade at Yll Morina is 132 students. In Rajmonda’s grade III class there were 34 students, 8-9 years old with 20 boys and 14 girls. The research was carried out in the period November 2011-June 2012. Prior to developing an action plan, a survey was distributed to teachers in order to identify the attitudes of teachers about word-problem solving in mathematics and practice directed at enabling students in this direction. The survey was distributed to 24 teachers at Yll Morina Elementary School. After the surveys were returned, the researchers, Rajmonda and Eda, interviewed the 34 students from III class to gain information about their attitudes in word-problem solving. Then students were tested to identify how the students actually performed word-problem solving tasks. The test consisted of four problems that students were required to solve. The data from the survey of the teachers along with the results of the first test given to students informed the development of the first action plan. To ensure the “trustworthiness” of this research, triangulation of the data along with collaboration with “critical-friends” was used (Creswell, 2008).
We implemented the first action plan in the III_3 classroom during November 2011. The first step was to administer a pre-test to students. This pre-test asked students to write definitions for 11 mathematical terms. We created several classroom activities to develop students' vocabulary. These interventions were intended to strengthen students’ abilities to understand and use different mathematical terms. We had observed that in many cases, students performed incorrect actions due either to a lack of understanding of the terms or an understanding of the expression in mathematical language. This is particularly problematic when it comes to solving word problems. To solve word problems, students should know mathematical vocabulary, understand mathematical concepts, and translate words from native language to mathematical (Sharma, 2001).

Every hour of teaching mathematics started with clarifying mathematical terms of the lesson in the instructional unit (Chard, 2003). These explanations continued in other phases of the lesson. The terms were explained extensively and students were instructed to write the vocabulary with special terms in their notebooks along with the definitions. These formed the students’ mathematics dictionary (Blessman & Myszczak, 2001; Brethouwer, 2008).

Rajmonda created a “word wall” of mathematics terms on a wall of the classroom. This was centrally located in the classroom so that students would be able to see and read them at any time (Burns, 2007). In many studies the word wall has been very effective for the development of mathematical vocabulary (Amen, 2006; Fogelberg et al., 2008; Georgius, 2008; Brethower, 2008).

During the implementation of the first action plan, other activities were employed to help strengthen students’ mathematics vocabulary. These activities included a game played by pairs of students. The game required students to explain words in mathematics word problems. The words came from the daily vocabulary list (Solomon, 2009). Another activity was the presentation of words using drawings. Students explained the words: addition, subtraction, multiplication and division through word explanations, presenting examples and illustrated with drawings using a worksheet called: What does it mean?

Summary of the first action plan:

- Pre-test on mathematical terms
- Clarification of everyday mathematical terms and students dictionary
- Word wall in classroom
- Word games played in pairs
- Two part diary – word explanation during problem solving
- Worksheet Activity: What does it mean?
- Post-test on mathematical terms (the same as the pre-test).

At the end of this plan, a post-test asked students to define the same words (Amen, 2006). We implemented a second action plan on formative assessment feedback strategies in March 2012. This was conducted from March-June 2012. Conducting formative assessment and providing feedback was the central intervention during this time. Feedback was given primarily by the teacher, but also by students for each other. Feedback was mainly provided in writing, but sometimes was also verbal. The purpose of the feedback was to improve the performance of students and to orient them to the proper procedures in word problem solving, highlighting potential errors in order to improve and clarify steps in problem solving. Students solved word
problems and for every problem solved feedback was given to improving the students’ performance, clarification and guidance in further work.

Summary of the second action plan:

- Provided writing and verbal feedback by the teacher
- Provided feedback by students for each other

At the end of each action plan tests were given to identify the impact of the action plans and the final test.

The teacher survey contained 11 questions, 9 used a Likert rating scale and two were open ended. Answers from the survey were analyzed by statistical method and the answers from open-ended questions first were read carefully to gain a general impression, identified text segments and then marked “codes” to describe the meaning of those segments (Creswell, 1998). The survey revealed that the teachers thought students have difficulty on understanding and solving word problems, they need guidance during problem solving and they lack necessary skills to solve word problems. Open-ended responses included comments such as:

- Students have difficulty to understand mathematical terms, do not understand the language of mathematics.
- Students do not have the patience to read mathematical problems, they see mathematics as numbers rather than words.

Teachers gave many reasons for the importance of developing students abilities for word problem solving: students will be more logical and will develop higher levels of thinking, will develop various skills they need for everyday life, will understand better concepts and mathematical content. Two of the respondents stated:

- Word problem solving develops students’ logic and promotes high levels of thinking.
- Enabling students to solve word problems help them to be better problem solvers in the future.

Of the 34 interviewed students, 30 students (88%) answered that they like to solve word problems, while 4 students (12%) stated the opposite.

**Table 1**

**Students attitudes for word problem solving**

<table>
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<th>Word problem solving</th>
<th>Positive attitudes</th>
<th>Negative attitudes</th>
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<tr>
<td></td>
<td>Word problems are fun. We like to solve word problems because we learn more. Word problems are interesting to be solved, there are always new things.</td>
<td>Difficulty in problem solving is misunderstanding of words. In word problems we have difficulties to find a solution. Determining the appropriate mathematical operation to solve problem is not easy.</td>
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</tbody>
</table>
Neutral attitudes
Word problem solving is the same as the solving other tasks in mathematics.

Results of the First Test with Students

Results from the first test showed that students have difficulties to solve word problems. Average scores achieved by students in this test is 10.74 (67% of the maximum score). Analysis of these results showed that students have difficulty in understanding mathematical terms.

Reduce the sum of numbers 39 and 47 with the product of the numbers 6 and 7.

Figure 1 Wrong solution (failure to understand the word zvogëloje or reduce)

Without an understanding of the meaning of the word reduce (zvogëloje), students failed to solve the problem accurately. This student, for example, increased the amount of numbers \((39 + 47)\) with the product of the numbers 6 and 7, while the problem required the opposite. This case showed that this student did not understand a key word in the problem, or did not pay attention to the meaning of the word, and therefore solved the problem incorrectly.

From this example and others, the researchers observed that students frequently did not understand the words: increase, decrease, double, \(x\) times more, \(y\) times less, and so on. Students often confused the words: less than or \(x\) times less, or more than, \(y\) times more, which leads to using the wrong operations during problem solving.

Figure 2. Wrong problem solution

There are 72 third grade poetry books in the school’s library, while in the library of the class there are eight times less poetry books than in the school’s library. How many poetry books are in the library of the class?
Students also found the wrong solution to the problems due to misunderstanding terms, in this case, the expression *eight times less*

Lack of students’ mathematical vocabulary is often accompanied by a lack in formulation of answers from students for solved problems. In many cases the problems are correctly calculated, but lacks a full written explanation of the answer.

Misunderstanding of the term *twice than a number* affects computation of twice by dividing. Noting the causes that impact the misunderstanding of mathematical words and the types of difficulties expressed by students in these and other cases, were found as part of the first action plan.

**Results of the Second Test with Students**

The second test was administered in order to identify the effects of the first action plan. Average score achieved by students was 12.56 (78% of the maximum score). The results showed that the first action plan has been influential in the development of students’ abilities to solve problems. 

*Table 2. Average scores of pre-test and post-test*

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<th></th>
<th>Pre-test</th>
<th>Post-test</th>
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<tr>
<td>Maximum Points</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Average scores achieved by students</td>
<td>7.68 (69.82% of the maximum score)</td>
<td>10.41 (94.64% of the maximum score)</td>
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</tbody>
</table>

Post-test results showed the improvement of student’s vocabulary and understanding of mathematical terms.
Students in some cases could not explain the different mathematical terms, so they explain them by using mathematical symbols.

Figure 5. Interpretation of mathematical concepts and difficulties in their explanation

The lack of understanding of mathematical terms leads to inaccurate explanations. When students were confused by mathematical words they used inappropriate actions to solve problems. After working with vocabulary student understood correctly mathematical words and gave proper explanation.

Figure 6. Wrong interpretation of the meanings product and quotient

The first action plan on the development of students’ mathematical vocabulary has helped develop students’ skills. Students who, at the beginning, had difficulties in understanding different mathematical terms, worked on the development of vocabulary and ultimately showed that they understood certain terms. These findings were realized after the results were collected and all student work was individually analyzed. While this first intervention led to improvements in students’ problem solving abilities, some students continued to have other difficulties in solving word problems. For this reason we implemented a second action plan.
Results of the Third Test with Students

The average score achieved by students in the third test was 13.97 (87.31% of the maximum score). The results confirmed the positive impact of the second action plan in the development of the students’ skills in problem solving.

A factory has had a request to produce 560 pencils in two days. The first day it produces 243 pencils, the second day it produces 87 pencils more. How many pencils the factory produces in two days? Did they achieve the goal to produce 560 pencils in two days.

Feedback through the explanations for correct problem solving for students showed improvements in their further work, because in this way students had opportunity to see their mistakes and have taken appropriate explanations for any uncertainty.

The Final Test Results

The final test results showed the positive impact of actions in developing students’ abilities in word problem solving. Average of final test score is 16.68 (91.75% of maximum score).
From the analysis of the results of four tests we saw positive changes in student performance. There was an increase in the number of problems solved correctly by students and in the average score for each test. The action plans have shown positive effects on students’ performance, after every action plan students results improved and the number of students who solved problems correctly increased. The actions in this research helped develop students’ abilities in word problem solving.

Word problem solving affects the development and application of student’s knowledge and abilities in mathematics, thus, it must be an integral part of the teaching and learning of mathematics. The difficulties that students experience when solving word problems can be improved by different strategies and activities, and students can be trained in this regard. Implementation of collaborative action research proved to be very efficient for the development of students’ skills and improving their work in word problem solving and changing teachers practice. Implementation of the action plans included in action research cycles resulted in achieving the purpose of the research and answered to the research questions. The research results showed that there is a direct link between problem solving and understanding of the vocabulary (Amen, 2006; Burns, 2007; Brethower, 2008; Georgius, 2006; McConnell, 2008). After working with mathematical vocabulary “students had higher achievement test scores” (Amen, 2006, p. 23). The research showed that the first action plan on the development of students’ mathematical vocabulary contributed to enabling students to better understand mathematical terms. Understanding mathematical terms impacted the results of word problem solving.

Amen (2006) showed that ‘students should write, visualize and act about mathematical vocabulary’ (p.24) and repeat it constantly. The findings from our current research project supported Amen’s results. Findings from Brethouer (2008) showed that the impact of vocabulary in learning mathematical concepts is related to a deeper understanding of them. McConell (2008) showed that students are better at problem solving when they understand the words in problem and the learning of vocabulary improves students’ understanding of mathematical concepts.

In addition to the impact of the learning vocabulary in problem solving, the research showed that the learning of vocabulary improved students’ communication and expression in mathematics and expanded their answers and reasoning for problem solving. Using the “word wall” and increasing students’ vocabulary retention has been an effective tool to help students in
memorizing various mathematical terms and utilizing them (Georgius, 2008; Brethower, 2008; Fogelberg, et al, 2010). A very useful strategy in this research was the feedback. Feedback provided clear information for students for correcting their work but also served as a guide in future work. Two-way communication between Rajmonda, the teacher, and students provided information for identifying the difficulties faced by students and at the same time it helped the discovery of the causes of these difficulties (Murchan, Shiel, & Vula, 2012). Feedback was very productive, especially when students gave useful information to each other as part of a peer review of the work.

An action research methodology was a very good way to develop student’s abilities in word problem solving and to improve classroom practice. Difficulties that students express in word problem solving can be overcome through various strategies and activities. It is important for students to have more opportunities to solve word problems, not only for accounting tasks but, more importantly, these problems allow them to connect mathematical concepts with real life problems. Developing students’ mathematical vocabulary helps students to understand the different words in mathematics, to express and communicate freely about mathematics, and to understand words and demands in word problems. Knowledge of various mathematical terms allows students to be better problem solvers. Students need clarification and guidance in their work, especially in word problem solving. Feedback was a useful strategy that enabled students to improve their work and be focused on further work. Finally, identification of students’ difficulties in learning, planning of activities according to the needs of students, the implementation of ‘action plans’ and reflecting on them can provide a quality education for our students.

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